

UNIVERSITY OF TORONTO

MAT 1300Y

Problem Set I

Due: Friday, Oct. 20, 2006

1. Show that $\text{card } X < \text{card } 2^X$ for any set X .
2. Let d be a metric on a set X . Show that $d' = d/(1 + d)$ also defines a metric on X . Verify that X with the topology coming from the metric d is homeomorphic to X with the topology coming from the metric d' .
3. Let A and B be subsets of a metric space. Define $d(A, B) = \inf\{d(a, b) \mid a \in A, b \in B\}$. Suppose A and B are closed with $A \cap B = \emptyset$. Is it possible that $d(A, B) = 0$?
4. Let A be a subset of a metric space X . Let V be a subset of X which is open in A . Show that there is an open set W in X such that $W \cap A = V$ and $\overline{W} \cap A$ is the closure of V in A .
5. Define functions $d_1, d_2, d_3 : \mathbb{R}^2 \rightarrow \mathbb{R}$ by $d_1(p, q) = \max\{|q_1 - p_1|, |q_2 - p_2|\}$, $d_2(p, q) = |q_1 - p_1| + |q_2 - p_2|$, and $d_3(p, q) = \sqrt{(q_1 - p_1)^2 + (q_2 - p_2)^2}$. Show that each of these functions is a metric on \mathbb{R}^2 and that the topologic spaces resulting from these metrics are homeomorphic.
6. Zariski topology
 - a) Consider the collection of subsets of \mathbb{C}^n which are the zero set of some finite collection of polynomials. Show that this collection of subsets form the closed sets of a topology on \mathbb{C}^n .
 - b) In the case $n = 1$, show that \mathbb{C} with this topology is not Hausdorff.
7. Show that a topological group is T_0 if and only if it is T_1 .
8.
 - a) Let A and B be closed subspaces of a topological group. Give an example to show that AB need not be closed.
 - b) (Qualifying Exam; Sept. 2002, # 1)
Let A and B be compact subspaces of a topological group. Show that AB is compact.