MAT 301 - Quiz 4 Solutions

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1) True or False. Briefly explain your answer.

a] There is a homomorphism $\phi : \mathbb{Z}_6 \to \mathbb{Z}_{20}$ for which $\phi(1) = 5$.

Answer: False, since $|5| = 4$ (in $\mathbb{Z}_{20}$) does not divide $|1| = 6$ (in $\mathbb{Z}_6$).

b] There exist two homomorphisms $\mathbb{Z}_2 \oplus \mathbb{Z}_2 \to \mathbb{Z}_4$.

Answer: True, take $\phi_1((a,b)) = 0$ for any (a,b), and take $\phi_2((a,b)) = 2a$ for any (a,b). Both are operation preserving, and the latter is well-defined, as $k \cdot 0 = 0$ and $2k \cdot 0 = 0$, while $k \cdot 1 = k \mod 2$ and $2k \cdot 1$ is 0 in $\mathbb{Z}_4$ if k is even and 2 in $\mathbb{Z}_4$ if k is odd.

2) Suppose ϕ is a homomorphism of $U(36)$ to some group for which $Ker\phi = \{1, 13, 25\}$ and $\phi(5) = 17$. Find the set $\phi^{-1}(17)$.

Answer: $\phi^{-1}(17) = 5\,Ker\phi = \{5{\cdot}1 \mod 36, 5{\cdot}13 \mod 36, 5{\cdot}25 \mod 36\} = \{5, 29, 17\}$.