



SIMULATIONS OF MHD TURBULENCE IN ISM

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ABSTRACT

We simulate supersonic, magnetized, compressible, isothermal turbulence using ideal MHD. To represent physical conditions within molecular clouds, our runs possess Mach number as high as 10 and have Alfvén Mach numbers ($\mathcal{M}_A \equiv \sigma/v_A$) around unity. Here, We study the rate of energy dissipation as well as the turbulence spectrum for runs with up to 128^3 cells. Further studies and larger runs are required to achieve the sufficient resolution and to separate physical properties of supersonic turbulence from numerical artifacts.

INTRODUCTION

Within molecular clouds, the line width corresponds to a one-dimensional velocity dispersion that scales with map size as

$$\sigma \approx 0.72 \left(\frac{R}{\text{pc}} \right)^{0.5} \text{ km s}^{-1} \quad (1)$$

[Lar81, SRBY87], and in many cases the Alfvén speed $v_A \equiv B/\sqrt{4\pi\rho}$ is inferred from Zeeman measurements to be comparable to σ [Cru99]. In contrast, CO rotational cooling maintains temperatures in the range 10-30 K and thermal sound speeds $c_s \sim 0.2 - 0.3 \text{ km s}^{-1}$ in the well-shielded regions to which star formation appears to be restricted [OMK⁺98, JDK04]. Relevant Mach numbers $\mathcal{M} \equiv \sigma/c_s$ vary from about unity, for the dense “cores” that form individual stars ($R \simeq 0.07 \text{ pc}$), to about 15 for the largest Milky Way clouds ($R \simeq 100 \text{ pc}$) [SRBY87].

Such a high Mach number seems to be at odds with theoretical estimates. Shock dissipation is the fastest. The broad line width suggests turbulent supersonic motions in molecular clouds, which has a slower dissipation rate than shocks. However, neither Kolmogorov theory for incompressible hydrodynamical turbulence nor the theory of subsonic, incompressible MHD turbulence ([SG94, GS95]) yields a small enough dissipation rate. The inferred high Alfvén speed v_A suggests the presence of a large magnetic field and the importance of MHD turbulence. Furthermore, the molecular cloud is a compressible medium. Following the spirits of [SOG98] and [Mac99], we perform simulations to study the dissipation rate of supersonic, compressible, isothermal MHD turbulence although previous numerical studies have not demonstrated the sufficiency of turbulence support.

NUMERICAL METHODS

Initial Condition: We have a cubic, periodic box of fluid which starts at rest, with uniform density ρ_0 , and threaded by a uniform magnetic field in the z-direction. The magnitude of the initial magnetic field is given by $b_0 \equiv B_0/\sqrt{4\pi} = \sqrt{2\rho_0 c_s^2/\beta}$ where $\beta \equiv P_{\text{gas}}/P_{\text{mag}}$. We use the isothermal approximation to model the gas dynamics in molecular clouds. In such a model, the sound speed c_s is constant in both space and time.

Advection Scheme: We solve the ideal MHD equation without artificial viscosity. Our code is an MPI version of a 2nd order, flux constraint transport, dimensional splitting, TVD code [PAW03]. We use operator splitting between advection and driving. In our TVD solver, we employ a smooth Minmod flux limit [Hir90].

Turbulence Driving Scheme: We use two methods of driving in this work which are distinguished by the forcing spectrum, the coherence time t_c of the driving force and by the manner in which it is normalized.

In either case, we update the fluid velocity on regular intervals

$$\delta t = 10^{-3} \frac{L}{c_s} \quad (2)$$

by an amount $\delta\mathbf{v}$. Each $\delta\mathbf{v}$ is an independent realization of a Gaussian random field with some prescribed power spectrum. In addition, we impose the conditions of divergence-free, which is achieved by taking the curl of a constructed vector potential, and zero-net-momentum for $\delta\mathbf{v}$. To avoid the use of parallel Fourier transform when constructing $\delta\mathbf{v}$, we generate a coarse (32^3) version of the global velocity perturbation field on each processor, and add the relevant region of $\delta\mathbf{v}$ to the local velocity field by interpolation.

Type I Driving – Type I driving mimics the methodology of [SOG98]. The spatial power spectrum of the driving velocity field is

$$|\widetilde{\delta\mathbf{v}}(k)|^2 \propto k^6 \exp\left(-8\frac{k}{k_{pk}}\right). \quad (3)$$

The velocity perturbation is normalized by $\phi(t)$, which is a *random* root of the quadratic energy equation ensuring an energy injection rate of $10^3\rho_0 L^2 c_s^3$ at every perturbation time step.

Such energy normalization implies that the magnitude of the imposed acceleration field is dependent on the current state of the fluid.

Type II Driving – Our second driving scheme differs in several ways from the first. Its power spectrum,

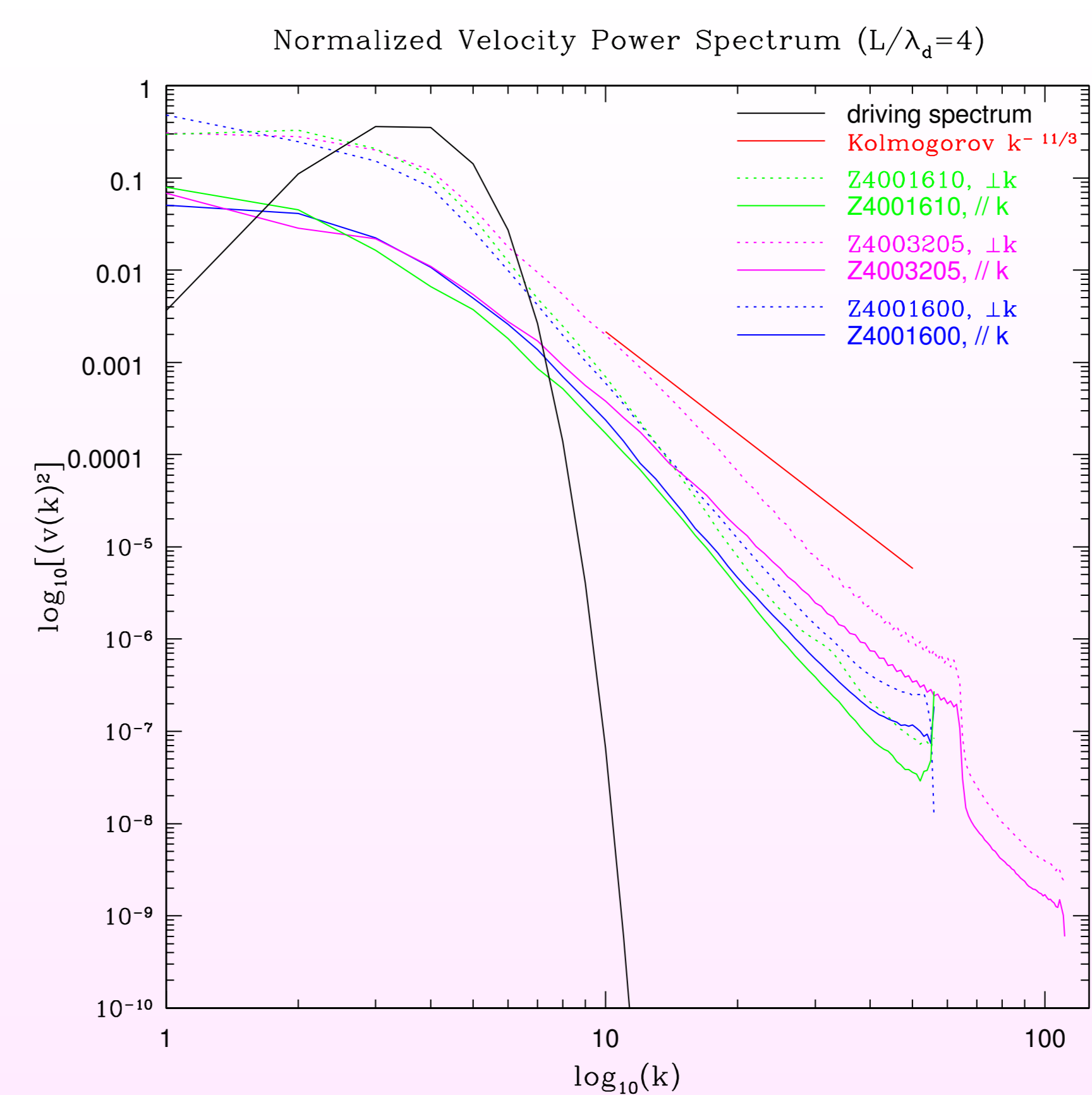
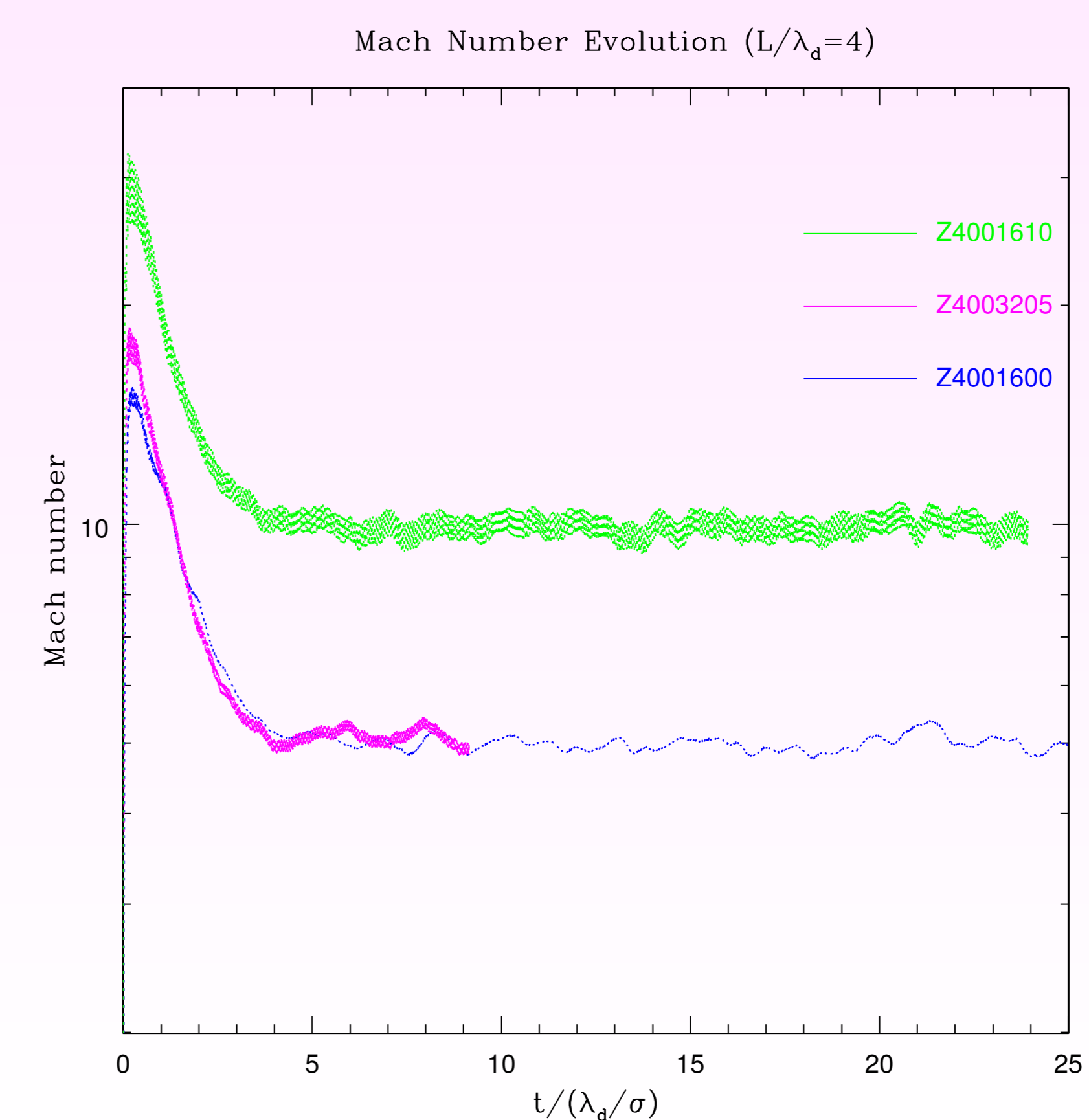
$$|\widetilde{\delta\mathbf{v}}(k)|^2 \propto k^6 \exp\left(-4\frac{k^2}{k_{pk}^2}\right), \quad (4)$$

peaks more narrowly around k_{pk} . We require that the velocity perturbation $\delta\mathbf{v}$ decoheres exponentially in the time scale t_c which we control to be of order λ_d/σ where $\lambda_d = 2\pi/k_{pk}$ and $\sigma = \sqrt{2E_k/(3M)}$.

In type II driving, the normalization factor ϕ is constant throughout a simulation run; it controls the average input power $\langle \dot{E} \rangle_t$.

PRELIMINARY RESULTS

We present here the Mach number evolution and the decomposed velocity power spectrums for some of our runs. The series code ‘Z’ indicates that the runs are performed with type II driving scheme. The first 2 digits after the series code ‘Z’ represents $\lambda_d/\Delta x$, the next 4 gives the value of L/λ_d , and the last two is v_A/c_s . Here, Δx is the grid size, and L the physical box size. Although we try to keep the Alfvén Mach number around unity, we adopt a Mach number of 5 for the run Z4001600. The simulations presented here are not well-resolved, with the highest resolution being only 128^3 .



CONCLUSION

As seen from the decomposed velocity power spectrums, the perpendicular component, i.e. the incompressible component, is dominant. The measured spectrums are steeper than Kolmogorov, which indicates the presence of shock dissipation. Shock dissipation might be inevitable in ‘supersonic turbulence’ if the term ‘turbulence’ remains valid under such circumstance. Larger simulations seem to have spectrums closer to Kolmogorov. However, we have not yet reached the resolution of convergence, and hence we do not know how much physical dissipation there really is. It is the goal of the current project to resolve the inertial range of supersonic, compressible MHD turbulence, and to quantify the rate of shock dissipation.

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