Lecture Slides for Wednesday Oct. 18 and Thursday Oct. 19

Is $f(x) = \tan(x)$ continuous on its domain?

In other words, is $f(x) = \tan(x)$ continuous at $x = a$ for all values of $a \in \text{Domain}(f)$?

Extreme Value Theorem: Suppose $f(x)$ is continuous on an interval of the form $[a, b]$. Then f attains both a maximum value M and a minimum value m.

Is this statement a bi-conditional statement ($A \iff B$) or a conditional statement ($A \implies B$)?
What do A and B represent?

For each of the following functions, decide which has a maximum y-value, a minimum y-value, and whether or not we can deduce those answer from the Extreme Value Theorem:

(a) $f(x) = x^2$ on [0, 3]

(b) $f(x) = x^2$ on [-1, 3]

(c) $f(x) = x^2$ on [-1, 3)

(d) $f(x) = x^2$ on (-1, 3]

Find a function that is continuous on $(0, 2]$ but has no minimum nor maximum value?

The Intermediate Value Theorem:

Let f be a function defined on the interval $[a, b]$ and K be any number in $[f(a), f(b)]$. IF

- $f(a) < K$ and $f(b) > K$
- (or if $f(a) > K$ and $f(b) < K$)
- f is continuous at all x-values in $[a, b]$

THEN $\exists c \in (a, b)$ such that $f(c) = K$.

Prove that $f(x) = x^3 - 4x + 2$ has at least two roots.

This marks the end of the material from Playlist 2. Would you like to request a topic for review before Test 1?

Definition: Let a be a real number and let f be a function defined at least on an open interval containing a. Then f is said to be differentiable at $x = a$ if:

$$lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

exists. If it does exist, we define $f'(a)$ to be the value of this limit. $f'(x)$ (read “f prime of x”) is the derivative of f.

Both of the following limits produce the same value. We sometimes use one as the definition of limit and sometimes the other. We can prove that they are equal to each other with a change of variable.

$$lim_{h \to 0} \frac{f(a + h) - f(a)}{h}$$

- Which substitution did the video use?
- Does it work? Check
- What does h represent geometrically? (i.e., what quantity is h, if the graph of f is sketched on an x-y axis?

Compute the derivative of:

- $f(x) = \sqrt{x}$
- $g(x) = \frac{1}{x}$
- $h(x) \cong 2$, the constant function with y-value equal to 2

A function is said to be <u>differentiable at $x = a$</u> if the derivative $f'(a)$ exists.

Is the absolute value function $f(x) = |x|$ differentiable at $x = 0$?

Prove your answer.

Is $f(x)$ continuous at $x = 0$?

So we know that continuity does not imply differentiability.

But the converse is true. State the converse.

Notation and Basic Derivatives (a class discussion involving the chalkboard)

Prove the quotient rule two ways:

(a) using first principles: $k'(x) = \lim_{h \to 0} \dfrac{k(x+h) - k(x)}{h}$

(b) using a combination of the product rule and chain rule.

For next week, please finish watching Playlist 3.