Lecture Slides for Wednesday Sept. 20 and Thursday Sept. 21

What is written on the page? We always understand math literally. The “spirit” of the proof might contradict the actual statement of the proof.

Are we given enough information to avoid guessing what the author meant?

Look for errors or missing information in these slides!
(And in your own writing!)

The goal is to be:

- as precise as possible when writing math
- as accurate as possible when reading math

Consider the following statement. Is this true? If yes, prove it. If not disprove it.

$\forall x, y \in \mathbb{R}, (x - y)^2 \geq 0$

Compare the proofs of

$$\forall x, y \in \mathbb{R},\ x^2 + y^2 \geq 2xy$$

Statements in math:

- Theorem
- Lemma
- Claim
- Proposition
- Axiom

Theorem: The product of two odd numbers is odd.

Write this statement with quantifiers and letters to represent numbers.

Write down a proof of this statement.

Is this a proof?

$x = 2a + 1$

$y = 2b + 1$

$xy = 2n + 1$

$(2a + 1)(2b + 1) = 2n + 1$

$4ab + 2a + 2b + 1 = 2n + 1$

$2(2ab + a + b) + 1$

$n = 2ab + a + b$

Can we improve or fix it?

Definition of "One-to-one":

A function f is said to be "injective" (also called one-to-one) if and only if $\forall x_1, x_2 \in \mathbb{R}, f(x_1) = f(x_2) \implies x_1 = x_2$.

Note: $\mathbb{R}$ can be replaced with any Domain for f.

Try to think of an example of such a function and sketch its graph.

Is there a way to tell from the graph whether f is one-to-one?

A function is one-to-one if every y values comes from at most one x value.

In other words, if $\exists x \in \text{Domain}(f)$ such that $y = f(x)$, then x is unique.

Here are some candidates. Which (if any) of the following functions f are one-to-one?

For this slide, suppose f is always a function defined on a non-empty domain D.

1. $f(x_1) \neq f(x_2)$.
2. $\forall x_1, x_2 \in D, x_1 \neq x_2, f(x_1) \neq f(x_2)$.
3. $\exists x_1, x_2 \in D$ such that $x_1 \neq x_2, \implies f(x_1) \neq f(x_2)$.
4. $\forall x_1, x_2 \in D,\ f(x_1) \neq f(x_2) \implies x_1 \neq x_2$.
5. $\forall x_1, x_2 \in D,\ f(x_1) \neq f(x_2) \implies x_1 = x_2$.
6. $\forall x_1, x_2 \in D$ such that $x_1 \neq x_2, \implies f(x_1) \neq f(x_2)$.

Is this statement true for any integer value of n?

$$4n < 2^n$$

Write down S_n, the value of n for the base case, and the induction hypothesis.

Why are we allowed to assume S_k?

Write down the statement of S_{k+1}. In order to prove that it is true, what do we have to show?

What are we allowed to use to prove S_{k+1} ? In other words, which facts, math statements, etc.?

What is required to prove the following statement?

$$\forall n \in \mathbb{Z}, n \geq 5, 4n < 2^n$$

Claim: For every even number n, the value of $n^2 + n$ can be written as a product of an odd number and an even number.

For next time:

- Watch videos 2.1 to 2.7 from Playlist 2
- Finish Problem Set 1
- Look at Problem Set 2