

Solutions to the Basic Algebra Test Questions:

1. $\frac{3}{4}$ is bigger than $\frac{5}{7}$. TRUE or FALSE

Solution: $\frac{3}{4} = \frac{21}{28}$ and $\frac{5}{7} = \frac{20}{28}$.

2. $\sqrt{120} = 2\sqrt{30}$. TRUE or FALSE

Solution: $\sqrt{120} = \sqrt{4 \times 30} = \sqrt{4}\sqrt{30} = 2\sqrt{30}$.

3. $\frac{x^2y^{-1} + 2x^7y^3}{2x^7y^3}$ simplifies to $x^2y^{-1} + 1$, if $xy \neq 0$. TRUE or FALSE

Solution: $\frac{x^2y^{-1} + 2x^7y^3}{2x^7y^3} = \frac{x^2y^{-1}}{2x^7y^3} + \frac{2x^7y^3}{2x^7y^3} = \frac{1}{2}x^{2-7}y^{-1-3} + 1 = \frac{1}{2x^5y^4} + 1$.

4. If $f(x) = \sqrt{3x^2 + 7x - 1}$, then $f(2) = 3$. TRUE or FALSE

Solution: $f(2) = \sqrt{3(2)^2 + 7(2) - 1} = \sqrt{25} = 5$.

5. $\sqrt[3]{-27} = 3$. TRUE or FALSE

Solution: $3 \times 3 \times 3 = 27$. $(-3) \times (-3) \times (-3) = -27$.

6. $\frac{3^x}{5} - \frac{2}{3^{-x}} = \frac{-3^{2+x}}{5}$ TRUE or FALSE

Solution: $\frac{3^x}{5} - \frac{2}{3^{-x}} = \frac{3^x 3^{-x}}{(5)3^{-x}} - \frac{2 \times 5}{(5)3^{-x}}$
 $= \frac{3^{x-x}}{(5)3^{-x}} - \frac{10}{(5)3^{-x}}$
 $= \frac{1-10}{(5)3^{-x}}$
 $= \frac{-(9)3^x}{5}$
 $= \frac{-3^2 3^x}{5}$
 $= \frac{-3^{2+x}}{5}$.

7. If $x \neq -2$, then $\frac{x^2 - 2x - 8}{x + 2} = x - 4$. TRUE or FALSE

Solution: $x^2 - 2x - 8 = (x - 4)(x + 2)$. So $\frac{x^2 - 2x - 8}{x + 2} = \frac{(x - 4)(x + 2)}{(x + 2)} = x - 4$.

8. If $a \neq 0$, $b \neq 0$, and $a \neq b$, then $\frac{\frac{1}{a} + \frac{1}{b}}{\frac{1}{a} - \frac{1}{b}} = \frac{a + b}{a - b}$. TRUE or FALSE

Solution: $\frac{1}{a} + \frac{1}{b} = \frac{b}{ab} + \frac{a}{ab}$ and $\frac{1}{a} - \frac{1}{b} = \frac{b}{ab} - \frac{a}{ab}$.

Therefore, $\frac{\frac{1}{a} + \frac{1}{b}}{\frac{1}{a} - \frac{1}{b}} = \frac{\frac{b+a}{ab}}{\frac{b-a}{ab}} = \frac{a + b}{b - a} = \frac{a + b}{-(a - b)}$.

9. If $a \neq 0$, $b \neq 0$, and $c \neq 0$, then $\frac{3a^2b^3c}{6bc^2} \div \frac{a^3b^3}{2abc} = 1$. TRUE or FALSE

Solution: $\frac{3a^2b^3c}{6bc^2} \div \frac{a^3b^3}{2abc}$

$= \frac{3a^2b^3c}{6bc^2} \times \frac{2abc}{a^3b^3}$

$= \frac{6a^3b^4c^2}{6a^3b^4c^2}$

$= 1$.

10. If $x > 0$, then $\frac{1}{(\sqrt{x^2 + x}) - x} = \sqrt{1 + \frac{1}{x}} + 1$. TRUE or FALSE

Solution: $\frac{1}{(\sqrt{x^2 + x}) - x} \times \frac{(\sqrt{x^2 + x}) + x}{(\sqrt{x^2 + x}) + x} = \frac{(\sqrt{x^2 + x}) + x}{x^2 + x - x^2}$

$= \frac{(\sqrt{x^2 + x}) + x}{x}$

$= \frac{\sqrt{x^2 + x}}{\sqrt{x^2}} + \frac{x}{x}$

$= \sqrt{\frac{x^2}{x^2} + \frac{x}{x^2}} + 1$

$= \sqrt{1 + \frac{1}{x}} + 1$.

Solutions to the Graphing Test Questions:

1. In the Cartesian Plane, the point $(-1, 3)$ is located above and to the left of the point $(-2, 1)$. TRUE or FALSE

Solution: The x -coordinate -2 is further to the left than the x -coordinate -1 .

2. The line with equation $3x + 15y = 35$ is parallel to the line with equation $x = -5y + 2$. TRUE or FALSE

Solution: The first equation rearranges to become $y = \frac{-1}{5}x + \frac{7}{3}$.

The other equation rearranges to become $y = \frac{-1}{5}x + \frac{2}{5}$.

Both lines have slope equal to $\frac{-1}{5}$ and are therefore parallel.

3. The line with equation $3x + 15y = 35$ is perpendicular to the line with equation $5x = y + 2$. TRUE or FALSE

Solution: The first equation rearranges to become $y = \frac{-1}{5}x + \frac{7}{3}$.

The second equation rearranges to become $y = 5x - 2$.

The perpendicular slope to a slope of $\frac{-1}{5}$ is 5 .

Therefore the lines are perpendicular to one another.

4. $y = 3x^2 + x + 1$ intercepts the x -axis twice. TRUE or FALSE

Solution: The quadratic formula tells us that the roots of $y = 3x^2 + x + 1$ are:

$$x = \frac{-1 \pm \sqrt{1^2 - 4(3)(1)}}{2(3)}.$$

But $\sqrt{1 - 12} = \sqrt{-11}$ does not exist since there is no real square root of a negative number.

Therefore $y = 3x^2 + x + 1$ has no roots and its graph never intercepts the x -axis.

5. The vertex of the parabola with equation $y = -2x^2 + 12x + 7$ is $(3, 25)$. TRUE or FALSE

Solution: Completing the square give us:

$$\begin{aligned}y &= -2x^2 + 12x + 7 \\&= -2(x^2 - 6x) + 7 \\&= -2(x^2 - 6x + 9 - 9) + 7 \\&= -2(x^2 - 6x + 9) + 18 + 7 \\&= -2(x - 3)^2 + 25.\end{aligned}$$

Therefore, the vertex of the parabola is $(3, 25)$.

6. If $\frac{x+3}{x-1} \leq 0$, then $-3 \leq x \leq 1$. TRUE or FALSE

Solution: In order for $\frac{x+3}{x-1} \leq 0$, the top and bottom must have opposite signs.

It is not possible for a single x -value to make the top negative while the bottom is positive. However, if $-3 \leq x < 1$, the top will be positive while the bottom is negative. It is important to note that if $x = 1$, the expression $\frac{x+3}{x-1}$ is undefined and so cannot be less than or equal to zero.

7. The graph of $y = \frac{1}{x + \pi}$ has no vertical asymptotes. TRUE or FALSE

Solution: The graph $y = \frac{1}{x + \pi}$ has a vertical asymptote at $x = -\pi$.

8. If the point $(3, 4)$ is on the graph of $f(x)$, then the point $(4, 3)$ cannot be on the graph of $f^{-1}(x)$. TRUE or FALSE

Solution: $f(3) = 4$. Therefore, $f^{-1}(4) = 3$.

9. The graph of $y = 2^{-x}$ has no points above the x -axis. TRUE or FALSE

Solution: The graph of $y = 2^x$ is entirely located above the x -axis. The graph of the equation $y = 2^{-x}$ can be obtained by reflecting the graph of $y = 2^x$ through the y -axis. In other words, it is also entirely located above the x -axis.

10. The circle with equation $3x^2 - 12x - 18y + 3y^2 = 0$ has center $(2, 1)$ and radius $\sqrt{5}$.

TRUE or FALSE

Solution: $3(x^2 - 4x) + 3(y^2 - 6y) = 0$

$$3(x^2 - 4x + 4 - 4) + 3(y^2 - 6y + 9 - 9) = 0$$

$$3(x^2 - 4x + 4) - 12 + 3(y^2 - 6y + 9) - 27 = 0$$

$$3(x - 2)^2 + 3(y - 3)^2 = 27 + 12$$

$$(x - 2)^2 + (y - 3)^2 = 13$$

Therefore, the circle has center $(2, 3)$ and radius $\sqrt{13}$.

Solutions to the Trigonometry Test Questions:

1. For all x , $\cos(-x) = \cos(x)$. TRUE or FALSE

Solution: The value of $\cos(\theta)$ can be found by starting at the positive x -axis and rotating counterclockwise about the origin along the unit circle. The x -coordinate of the endpoint on the unit circle is the value of $\cos(\theta)$. Likewise, the value of $\cos(-\theta)$ is obtained from the same method, only the rotation is clockwise. This results in two endpoints with the same x -coordinate. Therefore, $\cos(\theta) = \cos(-\theta)$ or $\cos(x) = \cos(-x)$.

2. $\sin \frac{\pi}{4} = \frac{1}{2}$. TRUE or FALSE

Solution: Consider a right-angled triangle with a non-hypotenuse side-length of 1 and another angle of measure $\frac{\pi}{4}$ radians. The third angle is $180^\circ - 90^\circ = 90^\circ$ or $\frac{\pi}{2}$ radians. The triangle must therefore be isosceles, meaning that both non-hypotenuse sides have length 1 and the hypotenuse has length c where $c^2 = 1^2 + 1^2 = 2$. Therefore, $\sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$.

3. $\tan\left(\frac{7\pi}{3}\right) = \sqrt{3}$. TRUE or FALSE

Solution: $\frac{7\pi}{3}$ has the same sin- and cos-values as $\frac{\pi}{3}$ but with opposite signs.

Therefore, $\sin\left(\frac{7\pi}{3}\right) = \frac{-\sqrt{3}}{2}$ and $\cos\left(\frac{7\pi}{3}\right) = \frac{-1}{2}$, making

$$\tan\left(\frac{7\pi}{3}\right) = \frac{\sin\left(\frac{7\pi}{3}\right)}{\cos\left(\frac{7\pi}{3}\right)} = \frac{\left(\frac{-\sqrt{3}}{2}\right)}{\left(\frac{-1}{2}\right)} = \frac{\sqrt{3}}{1} = \sqrt{3}.$$

4. For all θ , $\tan^2 \theta - \sec^2 \theta = -1$. TRUE or FALSE

Solution: $\cos^2 \theta + \sin^2 \theta = 1$. Dividing each term by $\cos^2 \theta$, we get:

$$\frac{\cos^2 \theta}{\cos^2 \theta} + \frac{\sin^2 \theta}{\cos^2 \theta} = \frac{1}{\cos^2 \theta}.$$

In other words, $1 + \tan^2 \theta = \sec^2 \theta$.

5. The radian measure of 36° is $\frac{\pi}{5}$. TRUE or FALSE

Solution: $36^\circ \times \frac{2\pi}{360^\circ} = \frac{\pi}{5}$.

6. If a triangle has sides of length 4 and 8 with an angle of $\frac{\pi}{3}$ between them, then one of the other angles must be $\frac{\pi}{4}$. TRUE of FALSE

Solution: Consider the $30^\circ - 60^\circ - 90^\circ$ triangle with side lengths 1, 2, and $\sqrt{3}$. If the triangle was enlarged by a factor of 2, then it would have side of lengths 4 and 8 with an angle of $\frac{\pi}{3} = 60^\circ$ between them. However, the other two angles would be $\frac{\pi}{6}$ and $\frac{\pi}{2}$.

7. If a triangle has sides with lengths a , b , and c and the angle across from side c is $\frac{\pi}{6}$, then $c^2 = a^2 + b^2 - \sqrt{3}ab$. TRUE or FALSE

Solution: Using cos-Law, we get that $c^2 = a^2 + b^2 - 2ab \cos C$, where $C = \frac{\pi}{6}$ and $\cos C = \frac{\sqrt{3}}{2}$.

8. The value of $\sin \theta + \cos \theta$ is the same for $\theta = 1095^\circ$ and $\theta = \frac{\pi}{12}$. TRUE or FALSE

Solution: 1095° has the same trig ratios as $1095 - 3 \times 360 = 15^\circ$. The degree measure for $\frac{\pi}{12}$ radians is $\frac{\pi}{12} \times \frac{360^\circ}{2\pi} = 15^\circ$.

9. The equation $\cos^2 \theta + \sin \theta + 5 = 0$ has at least one solution. TRUE or FALSE

$$\begin{aligned} \text{Solution: } (\cos^2 \theta) + \sin \theta + 5 &= (1 - \sin^2 \theta) + \sin \theta + 5 \\ &= -(\sin^2 \theta - \sin \theta - 6) \\ &= -(\sin \theta - 3)(\sin \theta + 2) \end{aligned}$$

However, if $-(\sin \theta - 3)(\sin \theta + 2) = 0$, then $\sin \theta = 3$ or -2 . But the values of $\sin \theta$ stay between -1 and 1 . Therefore, there are no solutions to the equation $\cos^2 \theta + \sin \theta + 5 = 0$.

10. $\cos^2(\theta) + \sin^2(-\theta) = -\cos(\pi)$ for all values of θ . TRUE or FALSE

Solution: $\cos(\pi) = -1$ and $\sin(-\theta) = -\sin(\theta)$.
Since $\sin^2(-\theta) = \sin(-\theta) \times \sin(-\theta)$, $\sin^2(-\theta) = \sin^2(\theta)$.
Also, $\cos^2(\theta) + \sin^2(\theta) = 1$.

Solutions to the Function Test Questions:

1. If $y = 3x + 6$ and $y = 0$, then $x = -3$.

TRUE or FALSE

Solution: $0 = 3x + 6 \Rightarrow 3x = -6$. Therefore, $x = -2$.

2. The equation $y = x^3 - 8$ has exactly one real root.

TRUE or FALSE

Solution: $x^3 - 8 = (x - 2)(x^2 + 2x + 4) = 0$

$x = 2$ is a solution of $x - 2 = 0$, but there are no solutions to $x^2 + 2x + 4 = 0$ since the left side of that equation is strictly positive.

3. $3^{x^2} = 9^{5x}$ is only true when $x = 0$ and $x = 10$.

TRUE or FALSE

Solution: This equation is equivalent to $3^{x^2} = (3^2)^{5x}$. In other words, $3^{x^2} = 3^{10x}$. Therefore, $x^2 = 10x$ or $x^2 - 10x = 0$. In factored form, this becomes $x(x - 10) = 0$. So $x = 0$ and 10 .

4. The functions $f(x) = \sqrt{3 - x}$ and $g(x) = \frac{1}{3 - x}$ have the same domain.

TRUE or FALSE

Solution: The domain of the first function is $\{x \in \mathbb{R} | x \leq 3\}$ while the domain of the second function is $\{x \in \mathbb{R} | x \neq 3\}$.

5. The range of the function $f(x) = x^2 - 8x + 7$ is $\{y \in \mathbb{R} | y \geq -9\}$.

TRUE or FALSE

Solution: $y = (x^2 - 8x + 16) - 16 + 7$. This is equivalent to the equation $y = (x - 4)^2 - 9$. The graph of this function has vertex $(4, -9)$. Since the graph of this equation is an upwards facing parabola, all the y values are higher than the y value of the vertex (i.e., -9).

6. The equation $x = \frac{2y - 1}{y}$ does not define a function of either variable.

TRUE or FALSE

Solution: Multiplying both sides by y , we get $xy = 2y - 1$. Bringing all y -terms to the left and then factoring, we get:

$xy - 2y = -1 \Rightarrow y(x - 2) = -1$. When $x \neq 2$, $y = \frac{-1}{x - 2}$ which defines y as a function of x .

7. The inverse of the function $f(x) = \frac{6x-1}{x+2}$ is TRUE or FALSE

$$f^{-1}(x) = \frac{1+2x}{6-x}.$$

Solution: If $y = \frac{6x-1}{x+2}$ then the inverse function is defined by $x = \frac{6y-1}{y+2}$. Solving for

$$y, \text{ we get: } x(y+2) = (6y-1) \Rightarrow xy+2x = 6y-1$$

$$xy-6y = -1-2x \Rightarrow y(x-6) = -1-2x$$

$$y = \frac{-(1+2x)}{-(6-x)}$$

8. If $f(x) = 7x+1$ and $g(x) = \frac{2}{x-2}$, then $f(g(4)) = 8$. TRUE or FALSE

Solution: $g(4) = \frac{2}{4-2} = 1$. $f(g(4)) = f(1) = 7(1)+1 = 8$.

9. If $h(x) = x^2+3x-1$ and $k(x) = \sqrt{x+3}$, then TRUE or FALSE

$$h(k(h(x))) = 3\sqrt{x^2+3x+2} + x^2+3x+1.$$

Solution: $k(h(x)) = \sqrt{(h(x))+3} = \sqrt{(x^2+3x-1)+3} = \sqrt{x^2+3x+2}$.

Similarly, $h(k(h(x))) = (k(h(x)))^2 + 3(k(h(x))) - 1 = (\sqrt{x^2+3x+2})^2 + 3\sqrt{x^2+3x+2} - 1$

$$= x^2+3x+2 + 3\sqrt{x^2+3x+2} - 1 = 3\sqrt{x^2+3x+2} + x^2+3x+1$$

10. It $|3x-4| > |2x+3|$, then either $x < \frac{1}{5}$ or $x > 7$. TRUE or FALSE

Solution: The expressions inside the absolute signs are equal to zero at $x = \frac{4}{3}$ and $x = \frac{-3}{2}$. The three cases arise from letting x be less than, between, and greater than these values.

Case 1: Let $x < \frac{-3}{2}$. Then the inequality becomes $-(3x-4) > -(2x+3)$

$3x-4 < 2x+3 \Rightarrow x < 7$. But we assumed that $x < \frac{-3}{2}$. So the we can conclude that all values of $x < \frac{-3}{2}$ are solutions.

Case 2: Let $\frac{-3}{2} < x < \frac{4}{3}$. Then $-(3x-4) > 2x+3$.

$4-3 > 2x+3x \Rightarrow 5x < 1 \Rightarrow x < \frac{1}{5}$. The solution to this case includes the solution to the previous case.

Case 3: Let $x > \frac{4}{3}$. Then $3x-4 > 2x+3$

Therefore $x > 7$.

Solutions to the Exponential and Logarithm Test Questions:

1. $\ln(e^2) = 2e$.

TRUE or FALSE

Solution: $\ln(e^2) = \ln e^2 = 2 \ln e = 2(1) = 2 \neq 2e$.

2. If $A > 0$ and $B > 0$, then $\ln(A - B) = \frac{\ln A}{\ln B}$.

TRUE or FALSE

Solution: $\frac{\ln A}{\ln B} = \log_B A \neq \ln(A - B)$.

3. $e^x = 0$ for exactly one value of x .

TRUE or FALSE

Solution: When $x \geq 0$, $e^x \geq 1$. When $x < 0$, the expression $0 < e^x < 1$.

Therefore, e^x is not equal to zero for any value of x .

4. $\log_2 0.25 = -0.125$.

TRUE or FALSE

Solution: $\log_2 0.25 = \log_2 \frac{1}{4} = \log_2 2^{-2}$.

If $\log_2 2^{-2} = z$, then $2^z = 2^{-2}$. So $z = -2 \neq -0.125$.

5. $\frac{\ln 1}{\ln x} = 0$ for all $x > 1$.

TRUE or FALSE

Solution: Since $x > 1$, $\ln x$ is a well-defined, non-zero value. $\ln 1 = 0$.

Therefore, $\frac{\ln 1}{\ln x} = 0$.

6. $\ln \sqrt{x^2 + 8x + 16} = \frac{1}{2} \ln(x + 4)$ for all values of x .

TRUE or FALSE

Solution: $\ln \sqrt{x^2 + 8x + 16} = \ln \sqrt{(x + 4)^2} = \ln(x + 4)$

7. If $3^{2x} = 9^{3x - \ln 2}$, then $x = \ln \sqrt{2}$.

TRUE or FALSE

Solution: $3^{2x} = (3^2)^{3x - \ln 2} \Rightarrow 3^{2x} = 3^{6x - 2 \ln 2}$

Therefore, $2x = 6x - 2 \ln 2$. So $-4x = -2 \ln 2$ and $x = \frac{1}{2} \ln 2 = \ln 2^{\frac{1}{2}} = \ln \sqrt{2}$.

8. $\ln(x^2 + 7x + 12) - \ln(x + 3) = \ln \frac{x + 4}{x + 3}$. TRUE or FALSE

Solution: $\ln(x^2 + 7x + 12) - \ln(x + 3) = \ln \frac{x^2 + 7x + 12}{(x + 3)}$
 $= \ln \frac{(x + 3)(x + 4)}{(x + 3)}$
 $= \ln(x + 4)$

9. The graphs of $y = \ln x$ and $y = e^x$ both have vertical asymptotes. TRUE or FALSE

Solution: $y = \ln x$ has a vertical asymptote at $x = 0$. However, the only asymptote of $y = e^x$ is the horizontal asymptote $y = 0$.

10. If $2^{4x-5} = 3^5$, then $x = \frac{5}{4}(\log_2 3 + 1)$. TRUE or FALSE

Solution: We want to write both sides of the equation as powers of 2. In order to do that we need to solve $2^z = 3$. This can be written as $z = \log_2 3$. Now the equation can be written as:

$$2^{4x-5} = (2^{\log_2 3})^5$$

$$2^{4x-5} = 2^{5(\log_2 3)}$$

$$4x - 5 = 5 \log_2 3$$

$$4x = 5 \log_2 3 + 5$$

$$4x = 5(\log_2 3 + 1)$$

$$x = \frac{5}{4}(\log_2 3 + 1)$$