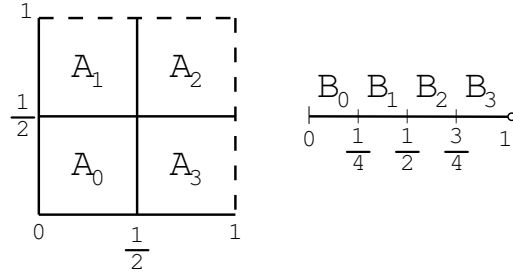


### The function that never was.

We want to construct a bijection between the points of the square  $[0, 1) \times [0, 1)$  and the half open interval  $[0, 1)$ . Let us partition these two sets into four congruent sets in the following way:



where  $A_i, B_i, i = 1, 2, 3$  are translations of  $A_0 = [0, \frac{1}{2}) \times [0, \frac{1}{2})$  and  $B_0 = [0, \frac{1}{4})$ , respectively. Suppose we had a bijection between  $[0, \frac{1}{2}) \times [0, \frac{1}{2})$  and  $[0, \frac{1}{4})$ . Then we would be done, because we could use this bijection to map the other three sets into each other.

We want to construct a bijection between the points of the square  $[0, \frac{1}{2}) \times [0, \frac{1}{2})$  and the half open interval  $[0, \frac{1}{4})$ . Suppose we had a bijection between  $[0, \frac{1}{2^2}) \times [0, \frac{1}{2^2})$  and  $[0, \frac{1}{4^2})$ . Then we would be done...