In-class Term Test 1

January 28, 6:10-7pm

No aids allowed.

Problem 1. (10 points) An element x of a ring R is called idempotent if $x^2 = x$.
a) Show that any non-trivial idempotent (i.e. $x \neq 0, 1$) in an unital ring is a zero divisor.
b) Show that if x is an idempotent then $1 - x$ is also an idempotent.

Problem 2. (10 points) Prove that any ring homomorphism $\phi : F \to R$, where F is a field is either injective or takes each element into 0.

Problem 3. (10 points) Let $I \subset \mathbb{Z}[x]$ be the ideal generated by a polynomial $x^2 + 1$. Show that the factor ring $\mathbb{Z}[x]/I$ is isomorphic to the ring of Gaussian integers $\mathbb{Z}[i]$. Is the ideal I prime? Is it maximal?