

## Homework 2

Due to 6pm, January 28, 2019

**Problem 1.** (15 points) Find all ideals  $I \subset \mathbb{Z}_{12}$ , so that the factor ring  $\mathbb{Z}_{12}/I$  is a field.

**Problem 2.** (15 points) Let  $I \subset \mathbb{R}[x]$  be an ideal generated by a polynomial  $x^2 + 1$ . Show that the factor ring  $\mathbb{R}[x]/I$  is isomorphic to the field of complex numbers  $\mathbb{C}$ .

Hint: Notice that the map  $\phi : \mathbb{R}[x] \rightarrow \mathbb{C}$  given by

$$\phi(a_0 + a_1x + \dots + a_kx^k) = a_0 + a_1i + \dots + a_ki^k$$

is a ring homomorphism. What is the kernel of  $\phi$ ?