Homework 1

Due to 6pm, January 21st 2019

Problem 1. Find all elements $x \in \mathbb{Z}_{14}$, that satisfy the equation

$$x^2 + 3x - 4 = 0.$$

Be careful, the ring $\mathbb{Z}_{14}$ has zero divisors!

Problem 2. Let $U \subset Mat_{2\times 2}(\mathbb{R})$ be a set of upper triangular matrices. In other words,

$$U = \left\{ \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} \text{ with } a, b, c \in \mathbb{R} \right\}.$$

a) Is U a subring of $Mat_{2\times 2}(\mathbb{R})$?
b) Is it a left ideal (i.e. $Mat_{2\times 2}(\mathbb{R}) \cdot U \subset U$)?
c) Is it a right ideal (i.e. $U \cdot Mat_{2\times 2}(\mathbb{R}) \subset U$)?
Justify your answer.

Problem 3. Let us define rings $\mathbb{Z}_3[i]$ and $\mathbb{Z}_3[\sqrt{2}]$ by

$$\mathbb{Z}_3[i] = \{a + bi \text{ with } a, b \in \mathbb{Z}_3 \text{ and } i^2 = -1\};$$

$$\mathbb{Z}_3[\sqrt{2}] = \{a + b\sqrt{2} \text{ with } a, b \in \mathbb{Z}_3\}.$$

Construct an isomorphism between $\mathbb{Z}_3[i]$ and $\mathbb{Z}_3[\sqrt{2}]$.