Summary of Curve Sketching

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Follow the following steps when sketching the graph of function $f(x)$.

1. x/y **intercepts.** Find the location of the x and y intercepts and plot them on the graph. The x-intercepts are the points $(a, f(a))$ such that $f(a) = 0$. That is, this is when f intercepts the x-axis. One obtains the value a by solving the equation $f(x) = 0$. The y intercept is the point $(0, f(0))$. That is, it is the point where f crosses the y-axis.

2. **Symmetry.** Look for symmetries of $f(x)$: even or odd? That is, $f(x) = f(-x)$ or $f(-x) = -f(x)$, respectively. Symmetries allow us to draw the graph of the function in the positive (or the negative axis) only. Then, we automatically know the shape of the function in the other part by appropriate reflections.

3. **Asymptotes.** Does the function have horizontal or vertical asymptotes? Horizonal asymptote can be detected by taking

$$\lim_{x \to \pm\infty} f(x) \, .$$

   If this limit exits, then f has a horizontal asymptote. Vertical asymptotes are usually detected at points where f is discontinuous or blows up.

4. **Interval on which $f(x)$ increases or decreases.** Use the first derivative test to 1) find intervals on which $f(x)$ is monotone, 2) identify critical points of $f(x)$, and 3) identify local and global extremum. If $f' > 0$ on an (a, b), then f is increasing. If $f' < 0$ on (a, b), then f is decreasing. A critical value x_0 is one such that $f'(x_0) = 0$ or $f(x_0)$ is not defined at the point. If x_0 is a critical value, the pair $(x_0, f(x_0))$ is called a critical point. A critical value x_0 is a local minimum if f' changes sign from negative to positive across x_0. Likewise, a critical point x_0 is a local maximum if f' changes sign from positive to negative across x_0.

5. **Concavity.** Use the second derivative test to obtain 1) intervals of concavity and 2) points of inflection. If $f'' > 0$ on (a, b), then f is concaving up on (a, b). Likewise, if $f'' < 0$, then f is concaving down. A point of inflection, x_0, is one at which f'' changes sign and f is continuous at x_0. In particular, at that point, $f''(x_0) = 0$.