FACULTY OF ARTS AND SCIENCE

University of Toronto

FINAL EXAMINATIONS, APRIL/MAY 2009

MAT 133Y1Y

Calculus and Linear Algebra for Commerce

PART A. MULTIPLE CHOICE

1. *[3 marks]*

If $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ is the solution of the system

$$\begin{array}{rcrcrcr} x & & & + & 3z & = & 2 \\ x & + & y & + & z & = & 2 \\ & & 2y & - & 3z & = & 1 \end{array}$$

then $x =$

Ⓐ -2

Ⓑ 0

Ⓒ -1

Ⓓ 2

Ⓔ 1

2. *[3 marks]*

Let $z = e^x + \ln(1 + x^2)$ and $x = \ln t + e^{(t-1)}$. When $t = 1$, $\dfrac{dz}{dt} =$

Ⓐ $2e$

Ⓑ 2

Ⓒ $2(e - 1)$

Ⓓ 1

Ⓔ $2(e + 1)$

3. *[3 marks]*

Let $y = \dfrac{(e^x+2)^{\frac{1}{2}}(x^3+2)^{\frac{1}{3}}}{(x^2+1)^{\frac{1}{4}}}$. Then $y' =$

Ⓐ $\dfrac{(e^x+2)^{\frac{1}{2}}(x^3+2)^{\frac{1}{3}}}{(x^2+1)^{\frac{1}{4}}}\left(e^x+3x^2-2x\right)$

Ⓑ $\dfrac{\frac{1}{2}(e^x+2)^{-\frac{1}{2}}e^x+\frac{1}{3}(x^3+2)^{-\frac{2}{3}}x^2}{(x^2+1)^{\frac{1}{2}}}$

Ⓒ $\dfrac{(e^x+2)^{\frac{1}{2}}(x^3+2)^{\frac{1}{3}}}{(x^2+1)^{\frac{1}{4}}}\left(\dfrac{e^x}{2(e^x+2)}+\dfrac{x^2}{x^3+2}-\dfrac{x}{2(x^2+1)}\right)$

Ⓓ $\frac{1}{2}(e^x+2)-\frac{1}{3}(x^3+2)-\frac{1}{4}(x^2+1)$

Ⓔ $\frac{1}{2}(e^x+2)^{-\frac{1}{2}}\dfrac{(x^3+2)^{\frac{1}{3}}}{(x^2+1)^{\frac{1}{4}}}+\dfrac{x^2}{(x^2+2)^{\frac{2}{3}}}\dfrac{(e^x+2)^{\frac{1}{2}}}{(x^2+1)^{\frac{1}{4}}}$

4. *[3 marks]*

When Newton's Method is applied to a particular function $f(x)$ with $x_0 = 1$, the following results are obtained from the first four iterations:

$x_1 = 2.6002$
$x_2 = 2.6013$
$x_3 = 2.6014$
$x_4 = 2.6014$

If f is a continuous function, then Newton's Method tells us:

Ⓐ that $f(x)$ has a root near $x = 2.6014$

Ⓑ that $f(x)$ has a critical point near $x = 2.6014$

Ⓒ that $f(x)$ has a local extremum at a such that $f(a) = 2.6014$

Ⓓ that $f(x)$ is increasing on the interval $(1, 2.6014)$

Ⓔ that $f(x)$ is concave up on the interval $(1, 2.6014)$

5. *[3 marks]*

$$\lim_{x\to 0} \frac{e^x - 1 - x - \frac{x^2}{2} - \frac{x^3}{6}}{x^4}$$

Ⓐ has no value

Ⓑ is equal to 0

Ⓒ is equal to $\dfrac{e}{4}$

Ⓓ is equal to $\dfrac{1}{24}$

Ⓔ is equal to e

6. *[3 marks]*

Let $f(x) = e^{x+\frac{1}{x}}$. Then on the interval $\dfrac{1}{2} \le x \le 2$, f has

Ⓐ no maximum and no minimum

Ⓑ a maximum at $x = 1$ and no minimum

Ⓒ a minimum at $x = \dfrac{1}{2}$ and a maximum at $x = 2$

Ⓓ a maximum at $x = \dfrac{1}{2}$ and $x = 2$ and no minimum

Ⓔ a minimum at $x = 1$ and a maximum at $x = \dfrac{1}{2}$ and $x = 2$

7. *[3 marks]*

If $\displaystyle\int_2^5 f(w)dw = 8$, then

$$\int_0^{21} \left[\frac{f(\sqrt{x+4})}{\sqrt{x+4}} + 4 \right] dx =$$

Ⓐ 8

Ⓑ 16

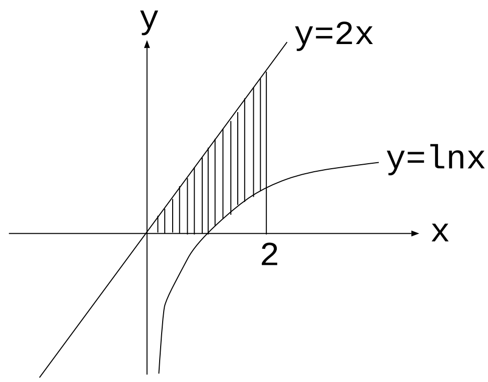
Ⓒ 92

Ⓓ 100

Ⓔ -8

8. *[3 marks]*

Given the following:



Which of the following does **not** represent the area of the shaded region?

(A) $\int_0^1 2x\,dx + \int_1^2 (2x - \ln x)\,dx$

(B) $\int_0^2 (2x - \ln x)\,dx$

(C) $\int_0^2 2x\,dx - \int_1^2 \ln x\,dx$

(D) $\int_{\ln 2}^4 \left(2 - \frac{y}{2}\right) dy + \int_0^{\ln 2} \left(e^y - \frac{y}{2}\right) dy$

(E) $\int_0^4 \left(2 - \frac{y}{2}\right) dy - \int_0^{\ln 2} (2 - e^y)\,dy$

9. *[3 marks]*

The rate at which the enrollment of a school is growing is directly proportional to the enrollment at any time. If the starting enrollment was 200 students and the enrollment a year later was 300 students then how many students will there be in 2 **more** years?

(A) 450

(B) 500

(C) 625

(D) 675

(E) 725

10. *[3 marks]*

If $\dfrac{dy}{dx} = \dfrac{y}{x}$ and $y = 4$ when $x = 2$, then when $x = 3$, $y =$

Ⓐ 5

Ⓑ 6

Ⓒ 7

Ⓓ 8

Ⓔ 9

11. *[3 marks]*

Let $g(x, y, z) = \dfrac{x + y + z}{x^2 + y^2 z + z^2}$.

When $(x, y, z) = (1, 1, 2)$, $\dfrac{\partial g}{\partial y} =$

Ⓐ $-\dfrac{9}{7}$

Ⓑ $\dfrac{9}{49}$

Ⓒ $-\dfrac{9}{49}$

Ⓓ $\dfrac{9}{7}$

Ⓔ 0

12. *[3 marks]*

If the joint demand functions for products A and B are

$$q_A = -{p_A}^2 - {p_B}^2 - 6p_A + 2p_B + 9$$
$$q_B = \ \ {p_A}^2 - {p_B}^2 - 2p_A - 4p_B + 7$$

then for the products to be competitive

Ⓐ $p_A < 1$ and $p_B > 1$

Ⓑ $p_A > 1$ and $p_B > 1$

Ⓒ $p_A < 1$ and $p_B < 1$

Ⓓ $p_A > 1$ and $p_B < 1$

Ⓔ no conditions will guarantee it

13. *[3 marks]*

If $f(x,y)$ is a function having first partial derivatives $\dfrac{\partial f}{\partial x} = x + 2y$ and $\dfrac{\partial f}{\partial y} = 2x + y$, and $x(t) = t^2,\ y(t) = t^3$, then when $t = 2$, $\dfrac{df}{dt} =$

Ⓐ 368

Ⓑ 216

Ⓒ 288

Ⓓ 192

Ⓔ 272

14. *[3 marks]*

If $(x^2 + y^2)z = z^2 + 6$, then when $(x, y, z) = (1, 2, 3)$, $\dfrac{\partial z}{\partial x} =$

Ⓐ 6

Ⓑ 3

Ⓒ 8

Ⓓ 4

Ⓔ 2

15. *[3 marks]*

If a and b are real constants,

$$\int_0^b \int_a^y 8xy\,dx\,dy =$$

Ⓐ $2b^4 - 2a^2b^2$

Ⓑ $b^4 - 2a^2b^2$

Ⓒ $2b^2y^2 - 4b^2ay$

Ⓓ $8b^2 - 8ab$

Ⓔ $\dfrac{8}{3}b^3 - 4ab^2$

PART B. WRITTEN-ANSWER QUESTIONS

B1. *[11 marks]*

(a) *[5 marks]*
Julius plans to save \$100,000 in 8 years by making 96 equal monthly deposits, the first deposit in 1 month. If his savings earn 6% compounded **quarterly**, how much should Julius deposit each month?

(b) *[6 marks]*
Julius follows the plan in part (a) but just after his 48th deposit, the interest rate on his savings drops to 3% compounded **monthly**. He will continue monthly deposits of the same size as before, but now must make **additional** deposits (beyond the 96 original planned). How many additional deposits must he make so that his savings equal or exceed \$100,000?

B2. *[11 marks]*

(a) *[8 marks]*
The NextStage Theatre Company finds that pricing their tickets at 50 dollars per ticket will sell 100 tickets. For every dollar decrease in price, they sell five extra tickets. What price will maximize the theatre company's revenue?

(b) *[3 marks]*
It costs NextStage 1000 dollars to put on a one-night production. Using the answer to part (a), find the maximum profit they can expect to make.

B3. *[13 marks]*

Evaluate the following integrals. Please give a numerical answer to four decimal places as your final answer, or show that the integral diverges.

(a) *[6 marks]*

$$\int_0^1 (e^x + x)^2 \, dx$$

(b) *[7 marks]*

$$\int_3^\infty \frac{4}{x^2(x-2)} \, dx$$

B4. *[10 marks]*

Let $f(x, y) = x^2 + 3y^2 - 2xy + x^3$. Find all critical points and for each critical point use the second derivative test to determine if it is a relative maximum, minimum, neither, or whether the test gives no information.

B5. *[10 marks]*

To fill an order for 150 units of its product a firm will distribute production between its two plants A and B. the total cost function is given by

$$c(q_A, q_B) = 4{q_A}^2 + 2q_Aq_B + 3{q_B}^2$$

when q_A and q_B are the number of units produced at plants A and B respectively. Use the method of Lagrange multipliers to determine how the output should be distributed so as to minimize costs. (You may assume the critical point does correspond to minimum cost.)

No other method besides Lagrange multipliers will be given any marks.

Solutions to April 2009 Exam, MAT133Y

PART A

1. ANSWER: Ⓒ

$$\begin{pmatrix} 1 & 0 & 3 & | & 2 \\ 1 & 1 & 1 & | & 2 \\ 0 & 2 & -3 & | & 1 \end{pmatrix}$$

$$\underset{\longrightarrow}{R_2 \to -R_1 + R_2} \begin{pmatrix} 1 & 0 & 3 & | & 2 \\ 0 & 1 & -2 & | & 0 \\ 0 & 2 & -3 & | & 1 \end{pmatrix}$$

$$\underset{\longrightarrow}{R_3 \to -2R_2 + R_3} \begin{pmatrix} 1 & 0 & 3 & | & 2 \\ 0 & 1 & -2 & | & 0 \\ 0 & 0 & 1 & | & 1 \end{pmatrix}$$

Hence $z = 1$

$y = 2z = 2$ which is not needed

and $x = 2 - 3z = -1$

2. ANSWER: Ⓔ

$$\left.\frac{dz}{dt}\right|_{t=1} = \left.\frac{dz}{dx}\right|_{x=1} \left.\frac{dx}{dt}\right|_{t=1} \quad \text{since } x = \ln 1 + e^0 = 1 \text{ when } t = 1$$

$$= \left(e^x + \frac{2x}{1+x^2}\right)\Big|_{x=1} \left(\frac{1}{t} + e^{t-1}\right)\Big|_{t=1}$$

$$= \left(e + \frac{2}{2}\right)\left(1 + e^0\right)$$

$$= 2(e+1)$$

3. ANSWER: Ⓒ

$$\ln y = \frac{1}{2}\ln(e^x+2) + \frac{1}{3}\ln(x^3+2) - \frac{1}{4}\ln(x^2+1)$$

$$\frac{1}{y}y' = \frac{e^x}{2(e^x+2)} + \frac{x^2}{x^3+2} - \frac{x}{2(x^2+1)}.$$

Multiplying by y gives Ⓒ

4. ANSWER: Ⓐ

Ⓐ that $f(x)$ has a root near $x = 2.6014$

5. ANSWER: Ⓓ

$$\begin{aligned}
&\lim_{x\to 0} \frac{e^x - 1 - x - \frac{x^2}{2} - \frac{x^3}{6}}{x^4} \quad \frac{0}{0} \\
&= \lim_{x\to 0} \frac{e^x - 1 - x - \frac{x^2}{2}}{4x^3} \quad \frac{0}{0} \\
&= \lim_{x\to 0} \frac{e^x - 1 - x}{12x^2} \quad \frac{0}{0} \\
&= \lim_{x\to 0} \frac{e^x - 1}{24x} \quad \frac{0}{0} \\
&= \lim_{x\to 0} \frac{e^x}{24} \quad = \quad \frac{1}{24}
\end{aligned}$$

6. ANSWER: Ⓔ

$$f'(x) = e^{x+\frac{1}{x}}\left(1 - \frac{1}{x^2}\right).$$

Since the interval is $[\frac{1}{2},\ 2]$, only $x = 1$ is a critical point. Since $f(x)$ is cont. on $[\frac{1}{2},\ 2]$ there must be a min and max and the only points to consider are

$$\begin{aligned}
x = \frac{1}{2} &\quad \text{where} \quad f(x) = e^{2.5} \\
x = 2 &\quad \text{where} \quad f(x) = e^{2.5} \\
\text{and} \quad x = 1 &\quad \text{where} \quad f(x) = e^{2}
\end{aligned}$$

Since $e^{2.5} > e^2$, f has a max at $x = \frac{1}{2}$ and $x = 2$ and a min at $x = 1$.

7. ANSWER: Ⓓ

$$\text{Let } u = \sqrt{x+4} \qquad du = \frac{1}{2\sqrt{x+4}}\,dx$$

$$\begin{aligned}
&\int_0^{21} \left[\frac{f(\sqrt{x+4})}{\sqrt{x+4}} + 4\right] dx = \\
&= \int_0^{21} \frac{f(\sqrt{x+4})}{\sqrt{x+4}}\,dx + \int_0^{21} 4\,dx \\
&= 2\int_2^5 f(u)du + 4x\Big|_0^{21} \\
&= 16 + 84 = 100
\end{aligned}$$

8. ANSWER: Ⓑ

Ⓑ $\displaystyle\int_0^2 (2x - \ln x)\, dx$

9. ANSWER: Ⓓ

$$\begin{aligned} N(t) &= 200e^{kt} \\ 300 &= 200e^{k} \\ e^k &= \frac{3}{2} \\ N(t) &= 200\left(\frac{3}{2}\right)^t \\ N(3) &= 200\left(\frac{3}{2}\right)^3 = 675 \end{aligned}$$

10. ANSWER: Ⓑ

$$\begin{aligned} \frac{dy}{y} &= \frac{dx}{x} \\ \ln|y| &= \ln|x| + C \\ |y| &= K|x| \quad \text{but } x \text{ and } y \text{ are positive} \\ \text{so} \quad y &= Kx \\ 4 &= 2K \quad \text{so} \quad K = 2 \\ y &= 2x \\ x = 3 &\Longrightarrow y = 6 \end{aligned}$$

11. ANSWER: Ⓒ

$$\begin{aligned} \frac{\partial g}{\partial y} &= \frac{(x^2 + y^2 z + z^2)\cdot 1 - (x + y + z)2yz}{(x^2 + y^2 z + z^2)^2} \\ &= \frac{(1 + 2 + 4) - (1 + 1 + 2)\cdot 2 \cdot 2}{(1 + 2 + 4)^2} \quad \text{at} \quad (x, y, z) = (1, 1, 2) \\ &= \frac{7 - 16}{7^2} \\ &= \frac{-9}{49} \end{aligned}$$

12. ANSWER: Ⓓ For products to be competitive,

$$\frac{\partial q_A}{\partial p_B} > 0 \quad \text{and} \quad \frac{\partial q_B}{\partial p_A} > 0$$

$$\frac{\partial q_A}{\partial p_B} = -2p_B + 2 > 0$$

$$2 > 2p_B$$

$$p_B < 1$$

$$\frac{\partial q_B}{\partial p_A} = 2p_A - 2 > 0$$

$$2p_A > 2$$

$$p_A > 1$$

13. ANSWER: Ⓔ

$$\frac{df}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$$

$$= (x + 2y) \cdot 2t + (2x + y) \cdot 3t^2$$

When $t = 2,\ x = 4$ and $y = 8$

$$\left.\frac{df}{dt}\right|_{t=2} = (4 + 16) \cdot 4 + (8 + 8) \cdot 3 \cdot 4$$

$$= 80 + 12 \cdot 16$$

$$= 272$$

14. ANSWER: Ⓐ

$$2xz + (x^2 + y^2)\frac{\partial z}{\partial x} = 2z\frac{\partial z}{\partial x}$$

$$2xz = (2z - x^2 - y^2)\frac{\partial z}{\partial x}$$

$$\frac{\partial z}{\partial x} = \frac{2xz}{2z - x^2 - y^2}$$

$$= \frac{2 \cdot 3}{6 - 1 - 4} = 6 \quad \text{at } (x, y, z) = (1, 2, 3)$$

15. ANSWER: Ⓑ

$$
\begin{aligned}
&\int_0^b 4x^2y\Big|_{x=a}^{x=y}\,dy\\
=&\int_0^b 4y(y^2-a^2)\,dy\\
=&\int_0^b (4y^3-4ya^2)\,dy\\
=&(y^4-2y^2a^2)\Big|_0^b\\
=&b^4-2a^2b^2
\end{aligned}
$$

PART B

B1.

(a) If $i =$ monthly rate, $(1+i)^3 = 1 + \dfrac{.06}{4} = 1.015$

$$100{,}000 = Rs_{\overline{96}|i}$$

$$R = \frac{100{,}000i}{(1+i)^{96} - 1} \qquad\qquad (1+i)^{96} = (1.015)^{32}$$

$$R = \frac{100{,}000[(1.015)^{\frac{1}{3}} - 1]}{(1.015)^{32} - 1}$$

$$\boxed{R \approx \$815.17}$$

note that $i = (1.015)^{\frac{1}{3}} - 1 = .004975206$ is not necessary to calculate.

(b) After 48 periods, he has

$$815.17S_{\overline{48}|i} = 815.17\frac{(1+i)^{48} - 1}{i}$$

$$= 815.17\frac{[(1.015)^{16} - 1]}{(1.015)^{\frac{1}{3}} - 1} \approx \$44{,}072.33$$

The monthly rate changes to $r = \dfrac{.03}{12}$, so n periods after the 48th he has

$$44{,}072.33(1+r)^n + 815.17s_{\overline{n}|r} = 100{,}000$$

$$44,072.33(1+r)^n + 815.17\frac{[(1+r)^n - 1]}{r} = 100{,}000$$

$$\left[44{,}072.33 + \frac{815.17}{r}\right](1+r)^n = 100{,}000 + \frac{815.17}{r}$$

$$\text{Remember : } r = \frac{.03}{12} \qquad n\ln(1+r) = \ln\left\{\frac{100{,}000 + \frac{815.17}{r}}{44{,}072.33 + \frac{815.17}{r}}\right\}$$

So $\boxed{n = 56.35}$

It takes $48 + 57 = 105$ periods in all, so $\boxed{\text{9 additonal ones}}$

B2.

(a) Let $x =$ no. of price decreases. $p = 50 - x \quad q = 100 + 5x$

$$R = pq = (50 - x)(100 + 5x) \qquad x \geq 0 \text{ (Could say } 0 \leq x \leq 50).$$
$$= -5x^2 + 150x + 5000$$
$$\frac{dR}{dx} = -10x + 150 = 0 \text{ when } x = 15, \text{ the \underline{only} crit pt.}$$

Argument for max

1) $\dfrac{d^2R}{dx^2} = -10 < 0$ for all values of x, so crit pt. is max

or 2) A parabola with negative coeff for x^2 has max at its vertex

or 3) $x < 15 \quad \frac{dR}{dx} > 0$ and $x > 15 \quad \frac{dR}{dx} < 0$ so R is increasing from $-\infty$ to 15 and decreasing forever after, hence max at $x = 15$

or 4) R is cont. on $[0, 50]$ so must have a max either at $x = 0$ or $x = 50$ or at crit pt. $x = 15$
$R(0) = 5000$
$R(50) = 0$
$R(15) = 35 \times 175 = 6125$ the biggest
So $x = 15$ and $\boxed{p = \$35}$ maximizes revenue.

(b) $\pi = R -$ cost
Since cost is constant, the optimal price does not change; neither does maximum revenue.

$$\pi = 6125 - 1000$$
$$= \boxed{\$5125}$$

B3.

(a)

$$\int_0^1 (e^x + x)^2\,dx$$
$$= \int_0^1 (e^{2x} + x^2 + 2xe^x)\,dx$$
$$= \left(\frac{1}{2}e^{2x} + \frac{x^3}{3}\right)\Big|_0^1 + 2\int_0^1 xe^x\,dx$$
$$= \left[\left(\frac{1}{2}e^2 + \frac{1}{3} - \frac{1}{2}\right] + 2\int_0^1 xe^x\,dx$$

Let $u = x \qquad dv = e^x dx \qquad du = dx \qquad v = e^x$

$$= \frac{1}{2}e^2 - \frac{1}{6} + 2\left[xe^x\Big|_0^1 - \int_0^1 e^x\,dx\right]$$
$$= \frac{1}{2}e^2 - \frac{1}{6} + 2\left(e - e^x\Big|_0^1\right)$$
$$= \frac{1}{2}e^2 - \frac{1}{6} + 2(e - e + 1)$$
$$= \frac{1}{2}e^2 + 2 - \frac{1}{6}$$
$$= \boxed{\frac{1}{2}e^2 + \frac{11}{6}}$$
$$= \boxed{5.5279}$$

(b)

$$\int_3^\infty \frac{4}{x^2(x-2)}\,dx$$
$$= \lim_{R\to\infty} \int_3^R \frac{4}{x^2(x-2)}\,dx$$
$$= \lim_{R\to\infty} \int_3^R \left[\frac{1}{x-2} - \frac{1}{x} - \frac{2}{x^2}\right]dx$$
$$= \lim_{R\to\infty} \left[\ln|x-2| - \ln|x| + \frac{2}{x}\right]_3^R$$
$$= \lim_{R\to\infty} \left[\ln\left(\frac{R-2}{R}\right) + \frac{2}{R} - \left(\ln\frac{1}{3} + \frac{2}{3}\right)\right]$$
$$= \boxed{\ln 3 - \frac{2}{3}} \approx .4319$$

$$\frac{4}{x^2(x-2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-2}$$
$$Ax(x-2) + B(x-2) + Cx^2 = 4$$
$$x = 2 \Rightarrow \quad 4C = 4 \Rightarrow \quad C = 1$$
$$x = 0 \Rightarrow \quad -2B = 4 \Rightarrow \quad B = -2$$

$x = 1$ for example $\Rightarrow$

$$-A - B + C = 4$$
$$-A + 2 + 1 = 4 \Rightarrow A = -1$$

B4.

$$f_x = 2x - 2y + 3x^2$$
$$f_y = 6y - 2x$$

$$\text{Crit pts}: f_y = 0 \quad \Rightarrow \quad x = 3y$$
$$f_x = 0 \quad \Rightarrow \quad 2(3y) - 2y + 3(3y)^2 = 0$$
$$4y + 27y^2 = 0$$
$$y(4 + 27y) = 0$$

$y = 0$: $\quad x = 3y \Rightarrow x = 0$ so $(0,0)$ is crit pt

$y \neq 0$: $\quad 4 + 27y = 0$

$y = -\frac{4}{27}$, $\quad x = 3y$ so $\left(-\frac{12}{27}, -\frac{4}{27}\right)$ is crit pt i.e. $\left(-\frac{4}{9}, -\frac{4}{27}\right)$

$$\boxed{\text{Crit pts} \quad (0,0), \qquad (-\frac{4}{9}, -\frac{4}{27})}$$

$$f_{xx} = 2 + 6x \quad f_{yy} = 6 \quad f_{xy} = -2$$
$$D = f_{xx}f_{yy} - (f_{xy})^2 = 6(2 + 6x) - 4$$
$$D = 8 + 36x$$
$$D(0,0) = 8 > 0 \text{ local extremum and} f_{xx} = 2 > 0$$

$$\boxed{(0,0) \text{ is relative min}}$$

$$D\left(-\frac{4}{9}, -\frac{4}{27}\right) = 8 - \frac{36 \times 4}{9} = -8 < 0$$

$$\boxed{\left(-\frac{4}{9}, -\frac{4}{27}\right) \text{ is neither a min nor a max}}$$

a saddle point.

B5.

$$q_A + q_B = 150$$

$$\mathfrak{L} = 4{q_A}^2 + 2q_Aq_B + 3{q_B}^2 - \lambda(q_A + q_B - 150)$$

$$\frac{\partial \mathfrak{L}}{\partial q_A} = 8q_A + 2q_B - \lambda = 0 \quad \frac{\partial \mathfrak{L}}{\partial \lambda} = 0 \Rightarrow q_A + q_B = 150$$

$$\frac{\partial \mathfrak{L}}{\partial q_B} = 2q_A + 6q_B - \lambda = 0$$

$$8q_A + 2q_B = \lambda$$

$$2q_A + 6q_B = \lambda \quad \text{and } q_A + q_B = 150$$

Fastest way is

$$8q_A + 2q_B = \lambda = 2q_A + 6q_B$$

$$6q_A = 4q_B$$

$$\frac{3}{2}q_A = q_B$$

$$q_A + \frac{3}{2}q_A = 150$$

$$\frac{5}{2}q_A = 150$$

$$\boxed{q_A = 60}$$

$$\boxed{q_B = 90}$$