FACULTY OF ARTS AND SCIENCE<br>University of Toronto

# FINAL EXAMINATIONS, APRIL/MAY 2007 <br> MAT 133Y1Y <br> Calculus and Linear Algebra for Commerce 

PART A. MULTIPLE CHOICE

1. [3 marks]

The system of equations

$$
\begin{aligned}
x+y+z+u+v & =1 \\
y+z+2 u & =2 \\
& =u+u+2 v
\end{aligned}
$$

has
(A) no solutions
(B) a unique solution
(C) infinitely many solutions with one parameter
(D) infinitely many solutions with two parameters
(E) infinitely many solutions with three parameters
2. [3 marks]

Let $h(x)= \begin{cases}\frac{x^{2}-1}{x-1} & \text { for } x \neq 1 \\ a-2 & \text { for } x=1\end{cases}$
Then $h(x)$ is continuous everywhere for $a$ equal to
(A) 4
(B) 3
(C) 1
(D) 2
(E) $\quad-2$
3. [3 marks]

If $f(x)=(\sqrt{x}+1)^{\frac{1}{x}+\ln x}, \quad \quad f^{\prime}(x)=$
(A) $(\sqrt{x}+1)^{\frac{1}{x}+\ln x}\left[\left(-\frac{1}{x^{2}}+\frac{1}{x}\right) \ln (\sqrt{x}+1)+\left(\frac{1}{x}+\ln x\right) \cdot \frac{1}{2(\sqrt{x}+1)}\right]$
(B) $e^{\left(\frac{1}{x}+\ln x\right) \ln (\sqrt{x}+1)}\left[-\left(\frac{1}{x^{2}}+\frac{1}{x}\right) \ln (\sqrt{x}+1)+\left(\frac{1}{x}+\ln x\right) \cdot \frac{1}{2 \sqrt{x}(\sqrt{x}+1)}\right]$
(C) $(\sqrt{x}+1)^{\frac{1}{x}+\ln x}\left[\left(-\frac{1}{x^{2}}+\frac{1}{x}\right) \ln (\sqrt{x}+1)+\left(\frac{1}{x}+\ln x\right) \cdot \frac{1}{(\sqrt{x}+1)}\right]$
(D) $e^{\left(\frac{1}{x}+\ln x\right) \ln (\sqrt{x}+1)}\left[\left(-\frac{1}{x^{2}}+\frac{1}{x}\right) \frac{\ln (\sqrt{x}+1)}{2 \sqrt{x}}+\left(\frac{1}{x}+\ln x\right) \cdot \frac{1}{2 x(\sqrt{x}+1)}\right]$
(E) $\quad(\sqrt{x}+1)^{\frac{1}{x}+\ln x}\left[\left(-\frac{1}{x^{2}}+\frac{1}{x}\right) \ln (\sqrt{x}+1)+\left(\frac{1}{x}+\ln x\right) \cdot \frac{1}{2 \sqrt{x}(\sqrt{x}+1)}\right]$
4. [3 marks]

The slope of the tangent line to the curve $x^{3}+3 y^{2}=4$ at $(1,1)$ is
(A) undefined
(B) $-\frac{1}{3}$
(C) $\frac{4}{3}$
(D) $\frac{3}{4}$
(E) $\quad-\frac{1}{2}$
5. [3 marks]

On the interval $\left(\frac{1}{2}, 2\right)$, the function $f(x)=e^{-\left(x^{2}+x^{-2}\right)}$ has
(A) a local minimum but no local maximum
(B) a local maximum but no local minimum
(C) a local maximum and a local minimum
(D) neither a local maximum nor a local minimum
(E) two local minima and no local maximum
6. [3 marks]
$\lim _{x \rightarrow \infty}\left(x^{2}+1\right)^{\frac{1}{x^{2}+2}}$
(A) does not exist
(B) $=e^{\frac{1}{2}}$
(C) $=e$
(D) $=0$
(E) $=1$
7. [3 marks]

When using the Trapezoidal Rule with the interval $[-2,6]$ divided into $n=4$ subintervals the approximate value of

$$
\int_{-2}^{6} \sqrt{1+x^{2}} d x
$$

is closest to
(A) 23.04
(B) 21.55
(C) 22.19
(D) 15.36
(E) $\quad 14.37$
8. [3 marks]
$\int_{-1}^{1} \frac{x^{\frac{2}{3}} d x}{\left(2+x^{\frac{5}{3}}\right)^{3}}=$
(A) $-\frac{1}{3}$
(B) $\frac{3}{5} \ln 27$
(C) $-\frac{64}{27}$
(D) $\frac{1}{5}$
(E) $\frac{4}{15}$
9. [3 marks]
$\int_{1}^{e} x \ln x d x=$
(A) $\frac{e^{2}-e+1}{2}$
(B) $\frac{1}{2}$
(C) $\frac{e^{2}}{2}$
(D) $\frac{e^{2}+1}{4}$
(E) 0
10. [3 marks]

$$
\int \frac{3 x-4}{(x-1)(x-2)} d x=
$$

(A) $\quad 3 \ln |(x-1)(x-2)|+C$
(B) $\quad 3 \ln \left|\frac{x-2}{x-1}\right|+C$
(C) $\quad 3 \ln \left|\frac{x-1}{x-2}\right|+C$
(D) $\quad \ln \left|(x-1)(x-2)^{2}\right|+C$
(E) $\quad \ln \left|\frac{x-1}{(x-2)^{2}}\right|+C$
11. [3 marks]
$\int_{8}^{\infty} \frac{1}{\sqrt[3]{x}} d x$
(A) diverges
(B) $=6$
(C) $=-6$
(D) $=4$
(E) $=-4$
12. [3 marks]

If $f(x, y, z)=\frac{x^{2} y^{3}}{z^{4}}, \quad f_{x y z}(2,-1,-2)=$
(A) $-\frac{3}{8}$
(B) $-\frac{3}{2}$
(C) $-\frac{1}{4}$
(D) $\frac{3}{2}$
(E) $\frac{3}{8}$
13. [3 marks]

If two products called $A$ and $B$ have the joint demand functions

$$
q_{A}\left(p_{A}, p_{B}\right)=1000-24 p_{A}+2 p_{A}^{2}-40 p_{B}+p_{B}^{2}
$$

and

$$
q_{B}\left(p_{A}, p_{B}\right)=2000+60 p_{A}-5 p_{A}^{2}-36 p_{B}-3 p_{B}^{2}
$$

then the products are competitive provided
(A) $\quad p_{A}<6$ and $p_{B}>20$
(B) $\quad p_{A}>6$ and $p_{B}>20$
(C) $p_{A}<6$ and $p_{B}<20$
(D) $\quad\left(p_{A}<6\right.$ and $\left.p_{B}<20\right)$ or $\left(p_{A}>6\right.$ and $\left.p_{B}>20\right)$
(E) $\quad p_{A}>6$ and $p_{B}<20$
14. [3 marks]

If $x(x+y+z)=y z$ then when $(x, y, z)=(1,2,3), \quad \frac{\partial x}{\partial z}=$
(A) $\frac{1}{6}$
(B) $\frac{1}{5}$
(C) $\frac{1}{7}$
(D) $\frac{1}{3}$
(E) $\frac{1}{8}$
15. [3 marks]

If the joint demand functions for the products $A$ and $B$ are given by

$$
q_{A}=\frac{4}{p_{A} \sqrt[3]{p_{B}}} \quad q_{B}=\frac{6}{p_{B} \sqrt{p_{A}}}
$$

and the joint cost function is given by

$$
C=q_{A}^{2}+2 q_{B}
$$

then when $p_{A}=4$ and $p_{B}=1, \quad \frac{\partial C}{\partial p_{A}}=$
(A) $-\frac{11}{4}$
(B) $-\frac{5}{4}$
(C) 2
(D) $\quad-4$
(E) $\frac{1}{4}$

## PART B. WRITTEN-ANSWER QUESTIONS

B1. [9 marks]
Tom owes Jerry two debts:

- $\$ 1000$ due now; and
- $\$ 3000$ plus interest at $5 \%$ compounded quarterly, due in 3 years.

They have agreed that the debts will be repaid in two payments:

- the first payment to be made in 4 years;
- the second payment to be one-third the amount of the first, to be made in 5 years.

What should the amounts of the first and second payments be, if money is worth $6 \%$ compounded semi-annually?

B2. [11 marks]
Find the values of $x$ and $y$ which minimize the function $f(x, y)=x y$ subject to the constraint $\frac{1}{x}+\frac{1}{y}=2$ by using Lagrange Multipliers (no need to verify your answer is a minimum and no marks will be given for any other method).

B3. [11 marks]
A bakery produces oatmeal and chocolate chip cookies. It costs $\$ 1 / \mathrm{kg}$ to make oatmeal cookies and $\$ 2 / \mathrm{kg}$ to make chocolate chip cookies. If the bakery sells $q_{o} \mathrm{~kg}$ of oatmeal cookies for $\$ p_{o} / \mathrm{kg}$ and $q_{c} \mathrm{~kg}$ of chocolate chip cookies for $\$ p_{c} / \mathrm{kg}$, then the joint demand functions are given by:

$$
q_{o}=100\left(p_{c}-p_{o}\right) \quad q_{c}=500+100\left(p_{o}-2 p_{c}\right)
$$

Find the prices $p_{o}$ and $p_{c}$ that result in maximum profit for the bakery.
(Be sure to verify that you actually get at least a local maximum by using the second derivative test for functions of two variables.)

B4. [12 marks]
Solve the following problems showing all your work.
(a) [6 marks]

If $\frac{d y}{d x}=x e^{y}$ and $y=0$ when $x=0$, what is $y$ when $x=1 ?$
(b) [6 marks]

If $\frac{d N}{d t}=N(1-N)$ and $N=\frac{1}{2}$ when $t=1$, what is $N$ when $t=2 ?$
[You may assume that $N(t)$ is always between 0 and 1.]

B5. [12 marks]
In both (a) and (b), your final answer should be a number rounded to two decimal places.
(a) [6 marks]

Find the present value of a 2 year continuous annuity at an annual rate of $6 \%$ compounded continuously if the rate of payment at time $t$ is $(5-t)$ million dollars per year.
(b) [6 marks]

Find $\int_{0}^{1} \int_{1}^{2}\left(2 y e^{x}-5 x e^{y}\right) d x d y$.

# Solutions to April 2007 Exam, MAT133Y PART A 

1. ANSWER: (D)

$$
\left(\begin{array}{lllll|l}
1 & 1 & 1 & 1 & 1 & 2 \\
0 & 1 & 1 & 2 & 0 & 2 \\
0 & 0 & 1 & 1 & 2 & 3
\end{array}\right)
$$

is already in row echelon form.

$$
\begin{gathered}
\# \text { var }-\# \text { non }- \text { zero rows }=\# \text { parameters } \\
5-3
\end{gathered}
$$

2. ANSWER: (A)

To be cont.,

$$
\begin{aligned}
\lim _{x \rightarrow 1} h(x) & =h(1)=a-2 \\
\lim _{x \rightarrow 1} \frac{x^{2}-1}{x-1} & =\lim _{x \rightarrow 1}(x+1)=2 \\
\text { so } \quad a-2 & =2 \\
a & =4
\end{aligned}
$$

3. ANSWER: ©

$$
\begin{aligned}
& \ln f=\left(\frac{1}{x}+\ln x\right) \ln (\sqrt{x}+1) \\
& \frac{1}{f} f^{\prime}=\left(-\frac{1}{x^{2}}+\frac{1}{x}\right) \ln (\sqrt{x}+1)+\left(\frac{1}{x}+\ln x\right) \frac{1}{\sqrt{x}+1} \cdot \frac{1}{2 \sqrt{x}}
\end{aligned}
$$

multiplying these by $f=(\sqrt{x}+1)^{\frac{1}{x}+\ln x}$, gets (E)
4. ANSWER: ©

$$
\begin{aligned}
3 x^{2}+6 y y^{\prime} & =0 \\
y^{\prime} & =-\frac{x^{2}}{2 y} \\
& =-\frac{1}{2} \quad \text { at } \quad(1,1)
\end{aligned}
$$

5. ANSWER: B

$$
\begin{aligned}
& f^{\prime}(x)=-e^{-\left(x^{2}+x^{-2}\right)}\left[2 x-\frac{2}{x^{3}}\right]=\frac{-2 e^{-\left(x^{2}+x^{-2}\right)}}{x^{3}}\left(x^{4}-1\right) \\
& f^{\prime}(x)=0 \text { only at } x=1,
\end{aligned}
$$

the only crit pt., on $\left(\frac{1}{2}, 2\right)$.

$$
\begin{aligned}
\frac{1}{2}<x<1 & \Rightarrow f^{\prime}>0 \\
(1,2) & \Rightarrow f^{\prime}<0
\end{aligned} \quad \bigwedge \quad \text { local max at } x=1
$$

and that's all, so (B)
6. ANSWER: (E)

$$
\begin{aligned}
y & =\left(x^{2}+1\right)^{\frac{1}{x^{2}+2}} \\
\ln y & =\frac{\ln \left(x^{2}+1\right)}{x^{2}+2} \quad \frac{\infty}{\infty} \text { as } x \rightarrow \infty \\
\lim _{x \rightarrow \infty} \ln y & =\lim _{x \rightarrow \infty} \frac{\frac{2 x}{x^{2}+1}}{2 x}=\lim _{x \rightarrow \infty} \frac{1}{x^{2}+1} \\
& =0 \\
\ln y & \rightarrow 0 \\
y & =e^{\ln y} \rightarrow e^{0}=1
\end{aligned}
$$

7. ANSWER: (A)

$$
\begin{aligned}
& x_{0}=-2 \\
& x_{1}=0 \\
& x_{2}=2 \\
& x_{3}=4 \\
& x_{4}=6
\end{aligned}
$$

$$
\begin{aligned}
\triangle_{x} & =\frac{6-(-2)}{4}=2 \\
T_{4} & =\frac{\triangle_{x}}{2}\left[y_{0}+2 y_{1}+2 y_{2}+2 y_{3}+y_{4}\right] \\
& =1[\sqrt{1+4}+2 \sqrt{1+0}+2 \sqrt{1+4}+2 \sqrt{1+16}+\sqrt{1+36}] \\
& \approx 23.037 \text { so }
\end{aligned}
$$

8. ANSWER: ©

Let

$$
\begin{aligned}
u & =2+x^{\frac{5}{3}} \quad x
\end{aligned}=-1 \Rightarrow u=1 \quad x=1 \Rightarrow u=3
$$

9. ANSWER: (D)

$$
\begin{aligned}
u & =\ln x \quad d v=x d x \\
d u & =\frac{1}{x} d x \quad v=\frac{x^{2}}{2} \\
\text { Integ. } & =\left.\frac{x^{2}}{2} \ln x\right|_{1} ^{e}-\frac{1}{2} \int_{1}^{e} x d x \\
& =\frac{e^{2}}{2}-\left.\frac{1}{2} \frac{x^{2}}{2}\right|_{1} ^{e} \\
& =\frac{e^{2}}{2}-\frac{1}{4}\left(e^{2}-1\right) \\
& =\frac{1}{4} e^{2}+\frac{1}{4}
\end{aligned}
$$

10. ANSWER: (D)

$$
\begin{aligned}
& \frac{3 x-4}{(x-1)(x-2)}=\frac{A}{x-1}+\frac{B}{x-2} \\
& A(x-2)+B(x-1)=3 x-4 \\
& x=1:-A=-1 \quad \text { so } \quad A=1 \\
& x=2: \quad B=2
\end{aligned}
$$

$$
\begin{aligned}
\text { Integral } & =\int\left[\frac{1}{x-1}+\frac{2}{x-2}\right] d x \\
& =\ln |x-1|+2 \ln |x-2|+C \\
& =\ln \left(|x-1||x-2|^{2}\right)+C
\end{aligned}
$$

11. ANSWER: (A)

$$
\begin{aligned}
& \lim _{R \rightarrow \infty} \int_{8}^{R} x^{-\frac{1}{3}} d x \\
& =\left.\lim _{R \rightarrow \infty} \frac{3}{2} x^{\frac{2}{3}}\right|_{8} ^{R} \\
& =\lim _{R \rightarrow \infty} \frac{3}{2}\left[R^{\frac{2}{3}}-4\right] \rightarrow \infty, \quad \text { so (A) }
\end{aligned}
$$

12. ANSWER: (D)

$$
\begin{aligned}
f_{x} & =\frac{2 x y^{3}}{z^{4}} \\
f_{x y} & =\frac{6 x y^{2}}{z^{4}} \\
f_{x y z} & =-\frac{24 x y^{2}}{z^{5}} \\
f_{x y z}(2,-1,-2) & =-\frac{24 \cdot 2 \cdot(-1)^{2}}{(-2)^{5}} \\
& =\frac{48}{32}=\frac{3}{2}
\end{aligned}
$$

13. ANSWER: (A)

$$
\begin{gathered}
\frac{\partial q_{A}}{\partial p_{B}}=-40+2 p_{B}>0 \Leftrightarrow p_{B}>20 \\
\frac{\partial q_{B}}{\partial p_{A}}=60-10 p_{A}>0 \Leftrightarrow p_{A}<6 \\
\quad \text { so } \quad p_{B}>20 \quad \text { and } \quad p_{A}<6
\end{gathered}
$$

14. ANSWER: ©

$$
\begin{aligned}
\frac{\partial x}{\partial z}(x+y+z)+x\left(\frac{\partial x}{\partial z}+1\right) & =y \\
\text { at }(1,2,3) \quad \frac{\partial x}{\partial z} \cdot 6+\frac{\partial x}{\partial z}+1 & =2 \\
7 \frac{\partial x}{\partial z} & =1 \\
\frac{\partial x}{\partial z} & =\frac{1}{7}
\end{aligned}
$$

15. ANSWER: B


$$
\begin{aligned}
\frac{\partial C}{\partial p_{A}} & =\frac{\partial C}{\partial q_{A}} \frac{\partial q_{A}}{\partial p_{A}}+\frac{\partial C}{\partial q_{B}} \frac{\partial q_{B}}{\partial p_{A}} \\
\frac{\partial C}{\partial q_{A}} & =2 q_{A} \\
& =\frac{8}{p_{A} \sqrt[3]{p_{B}}} \\
& =\frac{8}{4}=2 \text { at } p_{A}=4, p_{B}=1, \\
\frac{\partial C}{\partial q_{B}} & =2 \text { always } \\
\frac{\partial q_{A}}{\partial p_{A}} & =-\frac{4}{p_{A}^{2} \sqrt[3]{p_{B}}}=-\frac{4}{16}=-\frac{1}{4} \text { at } p_{A}=4, \quad p_{B}=1
\end{aligned}
$$

$$
\text { and } \quad \frac{\partial q_{B}}{\partial p_{A}}=-\frac{3}{p_{B} p_{A}^{3 / 2}}=-\frac{3}{8} \quad \text { at } p_{A}=4, \quad p_{B}=1
$$

$$
\frac{\partial C}{\partial p_{A}}=2\left(-\frac{1}{4}\right)+2\left(-\frac{3}{8}\right)=-\frac{5}{4}
$$

## PART B

B1.


We show the calculation at the end of the 5th yr (for convenience only):

$$
\begin{aligned}
1000(1.03)^{10}+3000\left(1+\frac{.05}{4}\right)^{12}(1.03)^{4} & =3 x(1.03)^{2}+x \\
x & =\frac{1000(1.03)^{10}+3000\left(1+\frac{0.5}{4}\right)^{12}(1.03)^{4}}{1+3(1.03)^{2}} \\
x & =1258.33 \\
3 x & =3775.00
\end{aligned}
$$

First payment $=\$ 3775.00$
2nd payment $=\$ 1258.33$

B2.

$$
\begin{aligned}
\mathcal{L} & =x y-\lambda\left(\frac{1}{x}+\frac{1}{y}-2\right) \\
\mathcal{L}_{x} & =y+\frac{\lambda}{x^{2}}=0 \rightarrow x^{2} y=-\lambda \\
\mathcal{L}_{y} & =x+\frac{\lambda}{y^{2}}=0 \rightarrow x y^{2}=-\lambda
\end{aligned}
$$

now divide the 2 nd equation by the 1 st :

$$
\begin{aligned}
& \frac{x^{2} y}{x y^{2}}=\frac{-\lambda}{-\lambda} \\
& \frac{x}{y}=1 \quad \text { and so } \quad x=y
\end{aligned}
$$

(The division is OK because $x=0$ and/or $y=0$ is forbidden by the constraint $\frac{1}{x}+\frac{1}{y}=2$, otherwise known as)
$\mathcal{L}_{\lambda}=0$
But if $x=y, \frac{1}{x}+\frac{1}{x}=2 \quad$ and $\quad x=1, \quad$ so $\quad y=1$
Note that there is a value of $\lambda$, namely $\lambda=-1$, but we don't really care about this.

B3. $\quad$ Let profit $=\pi$
Cost $=q_{0}+2 q_{c}$
Revenue $=p_{0} q_{0}+p_{c} q_{c}$

$$
\begin{aligned}
& \pi=\text { Revenue }- \text { Cost } \\
& \pi=q_{0} p_{0}+q_{c} p_{c}-q_{0}-2 q_{c}=q_{0}\left(p_{0}-1\right)+q_{c}\left(p_{c}-2\right) \\
& \pi=100\left(p_{c}-p_{0}\right)\left(p_{0}-1\right)+\left[500+100\left(p_{0}-2 p_{c}\right)\right]\left(p_{c}-2\right) \\
& \frac{\partial \pi}{\partial p_{c}}=100\left[\left(p_{0}-1\right)-2\left(p_{c}-2\right)+\left(5+p_{0}-2 p_{c}\right)\right]=100\left[2 p_{0}-4 p_{c}+8\right]=0 \\
& \frac{\partial \pi}{\partial p_{0}}=100\left[-\left(p_{0}-1\right)+\left(p_{c}-p_{0}\right)+\left(p_{c}-2\right)\right]=100\left[-2 p_{0}+2 p_{c}-1\right]
\end{aligned}
$$

Adding the 2 equations gives $100\left[-2 p_{c}+7\right]=0$
so $p_{c}=3.5$
but then using $\frac{\partial \pi}{\partial p_{0}}=0,-2 p_{0}+7-1=0$, so $p_{0}=3$.
The only critical point is $\quad p_{0}=\$ 3.00 \quad$ and $\quad p_{c}=\$ 3.50$

$$
\begin{aligned}
\frac{\partial^{2} \pi}{\partial p_{c}^{2}} & =-400 \\
\frac{\partial^{2} \pi}{\partial p_{0}^{2}} & =-200 \\
\frac{\partial^{2} \pi}{\partial p_{0} \partial p_{c}} & =200 \quad \text { always } \\
D=\frac{\partial^{2} \pi}{\partial p_{c}^{2}} \frac{\partial^{2} \pi}{\partial p_{c}^{2}}-\left(\frac{\partial^{2} \pi}{\partial p_{0} \partial p_{c}}\right)^{2} & =(-400)(-200)-(200)^{2} \\
& =40,000>0
\end{aligned}
$$

so our crit. pt. is a local extremum and $\frac{\partial^{2} \pi}{\partial p_{c}^{2}}<0$ so it is a local max.

B4.
(a)

$$
\begin{aligned}
\int e^{-y} d y & =\int x d x \\
-e^{-y} & =\frac{x^{2}}{2}+C . \text { At } x=0, y=0, \quad \text { this says } \\
-e^{0} & =0+C \text { so } C=-1 \\
-e^{-y} & =\frac{x^{2}}{2}-1
\end{aligned}
$$

When $x=1,-e^{-y}=\frac{1}{2}-1=-\frac{1}{2}$

$$
\begin{aligned}
e^{-y} & =\frac{1}{2} \\
e^{y} & =2 \\
y & =\ln 2 \approx .693
\end{aligned}
$$

(b)

$$
\begin{aligned}
& \int \frac{d N}{N(1-N)}=\int d t \\
& \int\left(\frac{1}{N}+\frac{1}{1-N}\right) d N=t+C \\
& \ln N-\ln (1-N)=t+C . \quad \text { At } \quad t=1, N=\frac{1}{2}, \quad \text { this says } \\
& \ln \frac{1}{2}-\ln \left(\frac{1}{2}\right)=1+C \\
& C+1=0 \quad \text { and } \quad C=-1
\end{aligned}
$$

Hence, $\quad \ln \frac{N}{1-N}=t-1$
When $\quad t=2, \ln \frac{N}{1-N}=1$

$$
\begin{aligned}
\frac{N}{1-N} & =e \\
N & =e-e N \\
N(1+e) & =e \\
N & =\frac{e}{1+e} \approx .731
\end{aligned}
$$

B5.
(a)

$$
\text { so } 7.58 \text { million dollars. }
$$

(b)

$$
\begin{aligned}
& =\int_{0}^{1}\left[2 y e^{x}-\frac{5 x^{2}}{2} e^{y}\right]_{x=1}^{x=2} d y \\
& =\int_{0}^{1}\left[\left(2 e^{2} y-10 e^{y}\right)-\left(2 e y-\frac{5}{2} e^{y}\right)\right] d y \\
& =\int_{0}^{1}\left[\left(2 e^{2}-2 e\right) y-\frac{15}{2} e^{y}\right] d y \\
& =\left[\left(2 e^{2}-2 e\right) \frac{y^{2}}{2}-\frac{15}{2} e^{y}\right]_{0}^{1} \\
& =\left(e^{2}-e-\frac{15}{2} e\right)-\left(-\frac{15}{2}\right) \\
& =\frac{15}{2}+e^{2}-\frac{17 e}{2} \\
& =-8.216 \\
& \text { so }-8.22
\end{aligned}
$$

$$
\begin{aligned}
& \text { P.V. }=\int_{0}^{2}(5-t) e^{-.06 t} d t \\
& u=5-t \quad d v=e^{-.06 t} d t \\
& d u=-d t \quad v=-\frac{e^{-.06 t}}{.06} \\
& =-\left.\frac{(5-t)}{.06} e^{-.06 t}\right|_{0} ^{2}-\frac{1}{.06} \int_{0}^{2} e^{-.06 t} d t \\
& =\frac{-3 e^{-.12}+5}{.06}+\left.\frac{1}{(.06)^{2}} e^{-.06 t}\right|_{0} ^{2} \\
& =\frac{5-3 e^{-.12}}{.06}+\frac{e^{-.12}-1}{(.06)^{2}} \approx 7.576 \cdots
\end{aligned}
$$

