University of Toronto
Faculty of Arts and Science

MAT133Y Final Exam

Instructor: Li Chen
Friday August 17, 2018, 2:00 pm – 5:00 pm
Duration: 3 hours

First name (please write as legibly as possible within the boxes)

Last name

Student ID

Instruction:

- **Allowed Aid:** TI-30X IIS calculator, to be supplied by the student. **No other aid is allowed.**
- Fill in this cover sheet. This exam contains 1 page for the multiple choice answer sheet and 18 pages for 6 problems. Once the exam begins, check to see if any pages are missing.
- For the multiple choice questions, you must record your answer by filling in the appropriate bubble on the answer sheet with a dark colored pen/pencil. **mark will only be awarded to answers on the answer sheet.**
- For the long answer problems, unless otherwise indicated, you are required to show your work on each problem on this exam. **You may not use any extra sheets and any attachment to the exam booklet will not be marked.**
- **Note that you will receive a maximum of 2 marks for an empty answer of a long answer question. If the questions has subsections, you will receive 1 mark per subsection for a maximum of 2 marks.**

Problem	Points	Score
1	45	
2	10	
3	14	
4	11	
5	10	
6	10	
Total:	100	

Part 1: Multiply Choice (40 marks)

Multiple Choice Questions

1. Multiple choice.

1 (3 points) An investment generates a return of $\$300,000$ with an initial investment of $\$205,000$ over a period of 3 years. The annual yield rate of the investment is closest to

A. 1.135%
B. 0.135%
C. 0.4634%
D. 13.5%
E. 23.4%

2 (3 points) If

$$\begin{cases} x+y &= 3 \\ 2x-z &= -1 \\ 3y-2z &= 0 \end{cases}$$

then $x =?$

A. 0
B. 1
C. 2
D. 3
E. 4

3 (3 points) Compute the following limit.

$$\lim_{x \to 0^+} (1+x)^{1/x^3}$$

A. 0
B. 1
C. e
D. $e^3 + 1$
E. ∞

4 (3 points) Suppose that $y^2 + x = e^{x+y} - 1$. Then $\frac{dy}{dx}$ at $x = y = 0$ is

A. -1
B. $-1/2$
C. 0
D. $1/2$
E. 1

5 (3 points) If the demand function is given by $p = 100q^{-1/2}$ and the cost is $c = 5q + 10$. Then the profit is maximized when $q =$?

A. 10
B. 50
C. 100
D. 150
E. 1000

6 (3 points) The point(s) of inflection of $y = 3x^5 - 5x^4 + 8x - 1$ is/are at

A. $x = 0$ and $x = 1$
B. $x = -1$
C. $x = 0$
D. $x = 1$
E. $x = -1$ and $x = 0$

7 (3 points) Let $x_0 = -1$. Use Newton's method to find an approximate solution, x_3, of $x^3 - x + 1 = 0$ by using 3 iterations. Then x_3 is closest to

A. -1.0000
B. -1.5000
C. -1.3478
D. -1.3252
E. -1.3247

8 (3 points) Compute the average of the function $f(x) = \frac{1}{x^2}$ on $[1, 10]$.

A. 1/10
B. 1/9
C. 9/10
D. 1
E. 9

9 (3 points) Compute the integral

$$\int_0^1 \frac{1}{x^2+3x+2}\,dx$$

A. 0
B. $\ln(4/3)$
C. $\ln(3/2)$
D. $4/3$
E. $3/2$

10 (3 points) Suppose that $f(0) = 1$ and $f(2) = 5$. Compute

$$\int_0^2 f'(x) + x\,dx$$

A. 2
B. 3
C. 4
D. 5
E. 6

11 (3 points) Let $f(x,y,z) = \frac{xyz}{x^2+y^2-z^2}$. Compute the value of f_z at $(1,1,0)$.

A. 1/4
B. 1/2
C. 1
D. 2
E. 0

12 (3 points) Let p_A and p_B be the prices of product A and product B respectively and q_A and q_B be their respective quantities. The demand functions for each are given by:

$$q_A = ke^{p_B} + 3p_A p_B - p_A^2 \text{ and } q_B = (1-k)p_A^2 - e^{p_B} - 2$$

These two products are competitive at all positive prices if

A. $k \geq 0$
B. $k = 0$
C. $k < 1$
D. $0 \leq k < 1$
E. No such k exists

13 (3 points) Let $f(x, y) = \ln(x^2 + y^2)$. For $(x, y) \neq 0$, compute

$$f_{xx} + f_{yy}$$

A. $\ln(x^2 + y^2)$
B. $x^2 + y^2$
C. 0
D. $2\ln(x) + 2\ln(y)$
E. $\frac{2x+2y}{x^2+y^2}$

14 (3 points) Suppose that $z = z(u, v, w)$, $u = u(x, y)$, $v = v(x, y)$, and $w = w(x, y)$. Moreover,

$$\begin{array}{ll}
z_u(1,2,0) = 1 & z_u(1,2,2) = 1 \\
z_v(1,2,0) = 2 & z_v(1,2,2) = -2 \\
z_w(1,2,0) = -1 & z_w(1,2,2) = 1 \\
 & \\
u(2,3) = 1 & u_x(2,3) = 2 \\
 & u_y(2,3) = -3 \\
 & \\
v(2,3) = 2 & v_x(2,3) = 1 \\
 & v_y(2,3) = 0 \\
 & \\
w(2,3) = 2 & w_x(2,3) = 0 \\
 & w_y(2,3) = 4
\end{array}$$

What is $\frac{\partial z}{\partial x} = \frac{\partial}{\partial x} z(u(x, y), v(x, y), w(x, y))$ at $x = 2$ and $y = 3$?
A. -2
B. -1
C. 0
D. 1
E. 2

15 (3 points) The critical point of

$$f(x,y) = x^2 + 2xy - y^2 - 2x - 10y + 25$$

is

A. $(3, -2)$, which is a saddle point
B. $(3, -2)$, which is a local max
C. $(3, -2)$, which is a local min
D. $(3, -2)$, but the second derivative test is inconclusive
E. There is no critical point.

Part 2: Long Answers (60 marks)

Show your work for full marks

2. Parts (a) and (b) below concern a 25 year Canadian mortgage of $\$500,000$ with monthly payments and annual interest at 2% compounded semiannually. In parts (a), (b), and (c), give answers to the nearest cent.

   (a) (3 points) Find the amount of each payment of the mortgage.

   (b) (4 points) What is the outstanding principal immediately after the first 10 years?

(c) (3 points) If a \$100 bond has 5 years to maturity with semiannual coupon payments at annual coupon rate of 6% and an annual yield rate of 3%, what is its market price today?

3. Compute the following integrals.

   (a) (7 points)

$$\int x \ln(x)\, dx$$

(b) (7 points)

$$\int_1^\infty \frac{3x-2}{x^3+2x^2}\,dx$$

4. Consider the graph of $x = y^2 + 2$ and $x = 5y - 4$.

   (a) (2 points) Find the points where the two graphs intersect

   (b) (5 points) Express as an integral the finite area bounded by those graphs. (Hint: do the names of the variable x and y matter?)

(c) (4 points) Find the area from part (b).

5. (10 points) Find an expression for y in terms of x if y satisfies the differential equation

$$\frac{dy}{dx} = (2x - 1)y$$

and $y = 2$ when $x = 1$.

6. (a) (4 points) Find and classify (i.e. relative max./relative min./saddle) all critical points of the function $f(x, y) = 5x^2 + 2xy - y^2 - 4x - 8y + 2$.

(b) (6 points) Use the method of Lagrange multiplier **only** to find the critical points of the joint cost function

$$c(q_A, q_B) = \frac{1}{2}q_A^2 + 100q_B + q_A q_B + \frac{1}{3}q_A^3 - 20$$

subject to the constraint

$$q_A + q_B = 10$$

[Show all your work. No marks will be given for any other method.]

1	A	B	C	D	E	21	A	B	C	D	E
2	A	B	C	D	E	22	A	B	C	D	E
3	A	B	C	D	E	23	A	B	C	D	E
4	A	B	C	D	E	24	A	B	C	D	E
5	A	B	C	D	E	25	A	B	C	D	E
6	A	B	C	D	E	26	A	B	C	D	E
7	A	B	C	D	E	27	A	B	C	D	E
8	A	B	C	D	E	28	A	B	C	D	E
9	A	B	C	D	E	29	A	B	C	D	E
10	A	B	C	D	E	30	A	B	C	D	E
11	A	B	C	D	E	31	A	B	C	D	E
12	A	B	C	D	E	32	A	B	C	D	E
13	A	B	C	D	E	33	A	B	C	D	E
14	A	B	C	D	E	34	A	B	C	D	E
15	A	B	C	D	E	35	A	B	C	D	E
16	A	B	C	D	E	36	A	B	C	D	E
17	A	B	C	D	E	37	A	B	C	D	E
18	A	B	C	D	E	38	A	B	C	D	E
19	A	B	C	D	E	39	A	B	C	D	E
20	A	B	C	D	E	40	A	B	C	D	E