Supplementary Questions for HP Chapter 5 and Sections 1 and 2 of the Supplementary Material

1. The manufacturer of a certain toy sells to retailers on either of the following terms:

   (i) Cash payment: 30% below recommended retail price

   (ii) Six months credit: 25% below recommended retail price.

(a) What is the effective rate of interest on the credit?

(b) The manufacturer is considering changing his credit terms. Credit for six months will no longer be available, but for three months credit a discount of 27.5% below the recommended retail price will be allowed. Does this discount offer a greater or lower effective rate of interest on credit than the previous six months credit?

2. A loan of $19,750 was repayable by an annuity of monthly payments (at the end of each month) for 20 years, on the basis of an effective interest rate of 9%. The lender had the right to alter the conditions of the loan at any time, and immediately after the 87th payment had been made, the effective rate was increased to 10%.

   The borrower was given the option of either

   (i) increasing the amount of the monthly payment, or

   (ii) extending the term of the loan (the monthly payment remaining the same).

(a) If the borrower chose option (i), find the increase in the value of the monthly payment.

(b) If the borrower chose option (ii), find the revised term of the loan, and the reduced amount of the final payment.

3. An investor purchased an annuity of payments annually for 20 years. The first payment, at the end of the first year, is $2,000 and the following payments increase by $100 each year. Based on a nominal interest rate of 4%, determine the purchase price. (Hint: consider the annuity as a sum of annuities of equal payments and use the formula for the sum of a geometric series.)

4. On November 1st, 1985 a man received three annuities, all payable by the same insurance company:

   (i) $200 annually payable on February 1st each year, the final payment being on February 1st, 2007.

   (ii) $80 quarterly payable on January 1st, April 1st, July 1st, and October 1st each year, the final payment being on January 1st, 2002.

   (iii) $15 monthly payable on the first day of each month, the final payment being on August 1st, 2004, the first payment on November 1st, 1985.

Immediately after receiving the first monthly payment due on November 1$^{\text{st}}$, 1985, the man requested these three annuities be combined, into a single annuity payable semi-annually on February 1$^{\text{st}}$ and August 1$^{\text{st}}$ each year, the final payment being on February 1$^{\text{st}}$, 2007. The request was granted.

Assuming all months are considered equal, find the semi-annual payments of the revised annuity, based upon an effective rate of 8% for all annuities.

5. A man receives interest payments at the end of each semi-annual period, for seven years, at a nominal annual rate of 5.5% on a $20,000 loan.

In addition, he will receive $110 in return for every $100 of the loan at the end of the seven years.

If the terms of the loan are altered so that repayment will be by a single sum of $23,362.67 in a number of years' time, calculate this time, if the repayment is based upon a nominal interest rate of 6% compounded semi-annually.

6. A man receives an annuity of equal semi-annual payments for the first five years, monthly payments of twice the initial value for the next five years, and quarterly payments of four times the initial value for the last five years.

The interest rate is an effective rate of 10%, compounded as often as the payments are given over the 15 years.

If the annuity has a present value of $2,049, find the initial value of the payments.

7. A loan of $16,000 is to be repaid by an ordinary annuity payable annually over ten years and calculated on the basis of a nominal interest rate of 8%. The terms of the loan provided that at any time the lender could alter the rate of interest, in which case the amount of the annual repayment would be revised appropriately.

(a) Find the initial amount of the annual repayment.

(b) Immediately after the fourth repayment was made, the annual rate of interest was increased to 10%. Find the revised amount of the (constant) annual repayment.

(c) Immediately after the seventh repayment was made, the annual rate of interest was reduced to 9%. There was no further change to the rate of interest. Find the final amount of the (constant) annual repayment and the effective rate of interest paid by the borrower on the completed transaction to within one-hundredth of a percentage point.

8. An annuity was purchased on September 1st, 1990 of installments on September 1st, 1991 and thereafter on January 1st, May 1st, and September 1st until May 1st, 1996, inclusive. The amount of the first installment was $1,000 and each subsequent installment was 5% greater than its predecessor.

    (a) Calculate the purchase price of the annuity on the basis of a nominal interest rate of 6%. (Hint: use the formula for the sum of a geometric series.)

    (b) Calculate the interest content of the seventh installment (interest paid on the present value on September 1, 1993 of the seventh and all remaining installments).

9. A loan of $9,880 was granted on July 10th, 1978. The installments are constant, monthly, and are for 25 years. The installments are calculated on the basis of an effective annual rate of interest of 7%, although payments are monthly. The first payment was on August 10th, 1978. Find:

(a) The monthly repayment

(b) The principal outstanding immediately after the repayment on March 10th, 1992.

(c) The principal to be repaid on October 10th, 1989.

(d) (i) the total principal to be repaid, and

    (ii) the total amount of interest to be paid

    in the monthly installments due between April 10th, 1996 and March 10th 1997 inclusive.

10. **(a)** A corporation plans to issue $1,000,000 in bonds to mature in 20 years paying interest at $11\frac{1}{2}$% per annum, payable semi-annually. The corporation will put on deposit, at the beginning of each year, sufficient funds to cover

    (i) the interest payments that must be made during the year, and,

    (ii) a (uniform) payment into a sinking fund annuity that will discharge their $1,000,000 obligation 20 years hence.

    Management estimates that it can earn 10% per annum compounded semi-annually on these deposits. Calculate the annual deposit required.

(b) The corporation later discovers that investors want a yield of 12%, payable semi-annually, forcing the corporation to sell the $11\frac{1}{2}$% bonds at a discount.

    (i) How much will be received from the sale of these bonds?

    (ii) State briefly how this change affects the amounts put on deposit in part (a).

11. A bond has a coupon rate of $5\frac{1}{2}$% payable quarterly on March 31st, June 30th, September 30th and December 31st. On September 30th, 1992, after the coupon payment, the bond had a price of $49.50 per $100 face value and had 20 years remaining to maturity.

(a) To within one-hundredth of a *percentage* point, find the quarterly yield rate.

(b) An investor bought $20,000 face value of this bond at the price and date quoted above. Exactly one year later, he sold his holding for $12,521.62. If the investor could have instead invested in a one year mortgage paying $1,100/ month at 12% compounded semi-annually for the same amount as the price he paid for the bonds, did the investor make the right choice? Note that the interest rate that the investor earns on the money received from the quarterly bond payments is the equivalent to answer (a), except compounded quarterly.