Thursday July 19, 2018
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Part 2: Long Answers (60 marks)

Show your work for full marks

2. (15 points) The equation $e^{3x} + e^{2y} = 10x - 4y + 2$ defines y implicitly in terms of x near the point $(0, 0)$. Find an expression for y' in terms of x and y and evaluate this expression at $(0, 0)$.

We (implicitly) differentiate both sides of the given equation:

$$3e^{3x} + 2e^{2y}\frac{dy}{dx} = 10 - 4\frac{dy}{dx}$$

5 points

Now solve for $\frac{dy}{dx}$:

$$2e^{2y}\frac{dy}{dx} + 4\frac{dy}{dx} = 10 - 3e^{3x}$$

$$\rightsquigarrow \quad \frac{dy}{dx} = \left(\frac{10-3e^{3x}}{2e^{2y}+4}\right)$$

5 points.

Now substitute $(x,y)=(0,0)$:

$$\left.\frac{dy}{dx}\right|_{(0,0)} = \frac{10-3e^{3\cdot 0}}{2e^{2\cdot 0}+4} = \frac{10-3}{2+4} = \frac{7}{6}$$

5 points.

4. Sketch the following curve

$$f(x) = x^3 - 3x^2 + 2.$$

by following the steps below.

(a) (3 points) Given that $f(1) = 0$, find x/y intercepts.

y-intercept: Substitute $x=0$ to get $f(0) = 2$, so y-intercept is $(0,2)$.

x-intercepts: Since $f(1)=0$, we know that $(x-1)$ divides $f(x)$.

Performing long division, we obtain:

$$\begin{array}{r} x^2-2x-2 \\ x-1 \overline{) x^3-3x^2+2} \\ x^3-x^2 \\ \hline -2x^2 \\ -2x^2+2x \\ \hline -2x+2 \\ -2x+2 \\ \hline 0 \end{array}$$

So we have

$f(x) = (x-1)(x^2-2x-2)$

The roots of x^2-2x-2 are

$x = \frac{2 \pm \sqrt{4+8}}{2} = 1 \pm \sqrt{3}$

So the x-intercepts of $f(x)$ are $1, 1+\sqrt{3}, 1-\sqrt{3}$

Marking Scheme: 1 point for correct y-intercept, 2 points for correct x-intercepts.

(b) (5 points) intervals on which f increases/decreases and find relative max. and relative min.

Differentiating $f(x)$, we have:

$f'(x) = 3x^2 - 6x = 3x(x-2)$, $\}$ 1 point

so $f'(x) > 0$ for $x \in (2, \infty) \cup (-\infty, 0)$ (increasing) $\}$ 1 point

and $f'(x) < 0$ for $x \in (0,2)$ (decreasing) $\}$ 1 point

and $f'(x) = 0$ for $x = 0, 2$ (relative min/max) $\}$ 1 point

Differentiating again, $f''(x) = 6x-6 = 6(x-1)$,

so $f''(0) = -6$ and $f''(2) = 6$.

Hence $x=0$ is relative max. and $x=2$ is relative min $\}$ 1 point

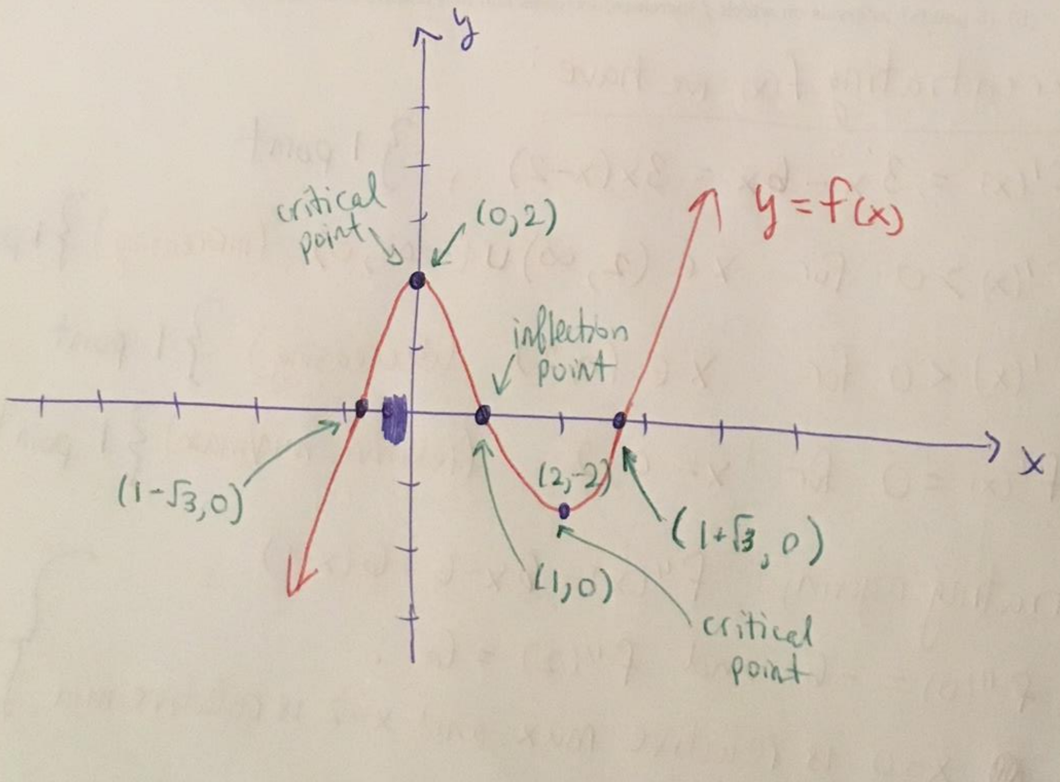
(c) (5 points) intervals of concavity and point(s) of inflection.

We know $f''(x) = 6(x-1)$ 2 points, so $f''(x) > 0$ for $x \in (1, \infty)$ (concave up) } 1 point

and $f''(x) < 0$ for $x \in (-\infty, 1)$ (concave down) } 1 point

and $f''(x) = 0$ when $x = 1$ (inflection point) } 1 point

(d) (2 points) Use the previous 3 parts to draw the curve. Be sure to label the x/y-intercepts, critical point(s), and point(s) of inflection.



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MC

1) B
2) B
3) A
4) A
5) D
6) B
7) A
8) D
9) D
10) E

Q 3)

$f(x) = x^4 - x - 3 \qquad x_0 = \frac{3}{2}, \qquad \overset{(2)}{f'(x) = 4x^3 - 1} \quad \& \quad \overset{(3)}{x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}}$

Step 1

$$f(x_0) = \left(\frac{3}{2}\right)^4 - \frac{3}{2} - 3 = \frac{81}{16} - \frac{24}{16} - \frac{48}{16} = \frac{9}{16} \quad (1)$$

$$f'(x_0) = 4\left(\frac{3}{2}\right)^3 - 1 = \frac{27}{2} - 1 = \frac{25}{2} = \frac{200}{16} \quad (1)$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = \frac{3}{2} - \frac{9}{16} \Big/ \frac{200}{16} = \frac{3}{2} - \frac{9}{200} = \frac{291}{200} = 1.455 \quad (3)$$

Step 2

$$f(x_1) = \left(\frac{291}{200}\right)^4 - \frac{291}{200} - 3 = 0.0268 \quad (1)$$

$$f'(x_1) = 4\left(\frac{291}{200}\right)^4 - 1 = 11.3211 \quad (1)$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1.455 - \frac{0.0268}{11.3211} = 1.4526 \quad (3)$$

$$\Rightarrow \boxed{x_2 = 1.4526}$$

Q5)

a)

$$\int \frac{x}{x+1}\,dx \overset{\text{using hint}}{=} \int \frac{x+1-1}{x+1}\,dx \quad (1)$$

$$= \int \frac{x+1}{x+1} - \frac{1}{x+1}\,dx = \int \frac{x+1}{x+1}\,dx - \int \frac{1}{x+1}\,dx = \int dx - \int \frac{1}{x+1}\,dx \quad (2)$$

$$= x + C - \log(x+1) + D \quad (3)$$

$$\Rightarrow \boxed{y = x - \log(x+1) + C} \quad (1)$$

b)

$$\int 4x^3 + 2e^{x-1}\,dx = \int 4x^3\,dx + \int 2e^{x-1}\,dx = x^4 + 2e^{x-1} + \underset{y(0)}{C} \quad (4)$$

$\Rightarrow y(x) = x^4 + 2e^{x-1} + y(0)$

plug in $y(1) = 2$

$\Rightarrow 2 = y(1) = 1^4 + 2e^{1-1} + y(0) \Rightarrow 2 = 1 + 2 + y(0) \Rightarrow y(0) = -1$ (3)

$$\Rightarrow \boxed{y = x^4 + 2e^{x-1} - 1} \quad (1)$$

Q5a) alternate solution

$$\int \frac{x}{x+1}\,dx \quad (1) \qquad \begin{matrix} \text{Cof } V \\ x+1=u \\ dx=du \end{matrix}$$

$$= \int \frac{u-1}{u}\,du = \int du - \int \frac{1}{u}\,du \quad (3)$$

$$= u + c + \log(u) + 0 \quad (2)$$

$$= x+1 + \log(x+1) + C \quad (1)$$