# Faculty of Arts and Science <br> University of Toronto <br> MAT133Y Term Test 3 <br> Thursday July 19, 2018, 7:10 pm - 9:00 pm <br> Duration - 110 minutes 

## Surname:

$\qquad$

## Given Name:

## Student Number:

Tutorial Section:

Allowed Aid: A TI-30X IIS calculator, to be supplied by the student. No other aid is allowed.
Instruction:

- Fill in all required information on this cover sheet and the multiple choice answer sheet on the last page of your exam. DO NO TEAR THE ANSWER SHEET. MARK WILL ONLY BE AWARDED TO ANSWERS ON THE ANSWER SHEET!
- This exam contains 14 pages (including this cover page) and 5 problems. Once the exam begins, check to see if any pages are missing.
- Unless otherwise indicated, you are required to show your work on each problem on this exam. If you need more space, use the back of the pages; clearly indicate when you have done this.
- On the written portion of the test, BLANK ANSWER WILL RECEIVE 2 MARKS PER QUESTION or 1 MARK PER SUBQUESTION. Leave a question or a subquestion blank if you do not know how to do it. You will not be entitled to the free mark if you attempted the question.
- Questions are not order in increasing order of difficulties. Be sure to read through all the problems and plan your time well.

| Section | Time | Location | Instructor |
| :--- | :--- | :--- | :--- |
| TUT0101 | T $14-15$ | BA 2139 | Tristan Milne |
| TUT0101 | R $14-15$ | BA 2139 |  |
| TUT0102 | T $14-15$ | BA 1230 | Kai Wang |
| TUT0102 | R $14-15$ | BA 1230 |  |
| TUT0103 | T $14-15$ | BA 2195 | Dmitri Chouchkov |
| TUT0103 | R $14-15$ | BA 2195 |  |
| TUT0201 | T $15-16$ | BA 2139 | Tristan Milne |
| TUT0201 | R $15-16$ | BA 2139 |  |
| TUT0202 | T $15-16$ | BA 1230 | Kai Wang |
| TUT0202 | R $15-16$ | BA 1230 |  |


| Problem | Points | Score |
| :---: | :---: | :---: |
| 1 | 40 |  |
| 2 | 15 |  |
| 3 | 15 |  |
| 4 | 15 |  |
| 5 | 15 |  |
| Total: | 100 |  |

## Part 1: Multiply Choice (40 marks)

## Multiple Choice Questions

1. Multiple choice.

1 (4 points) Find $\int \frac{1}{x} d x$
A. $0+C$
B. $\ln |x|+C$
C. $x+C$
D. $-x^{-2}+C$
E. $1+C$

2 (4 points) Critical point(s) of $f(x)=x\left(x^{2}-3\right)$ is/are
A. $0, \pm \sqrt{3}$
B. $\pm 1$
C. $0, \pm 1$
D. $\pm 3$
E. 0

3 (4 points) Find

$$
\int \frac{x}{\sqrt{x^{2}+2}}
$$

explicitly.
A. $\sqrt{x^{2}+2}+C$
B. $x \sqrt{x^{2}+2}+C$
C. $\frac{x}{\sqrt{x^{2}+2}}+C$
D. $2\left(x^{2}+2\right)^{-3 / 2}+C$
E. $\left(x^{2}+2\right)^{-1 / 2}+C$

4 (4 points) If $2 x^{2}+y^{2}=4$, then $\frac{d y}{d x}$ at $(0,2)$ is
A. 0
B. $\frac{1}{2}$
C. 1
D. 4
E. 8

5 (4 points) If $f(x)=x^{x}$, then $f^{\prime}(x)=$
A. $(\ln x) x^{x}$
B. $x \cdot x^{x-1}$
C. $x^{x}$
D. $x^{x}(\ln (x)+1)$
E. $e^{x \ln x}$

6 (4 points) Let

$$
y=\frac{2 x^{3}+6}{x^{2}}
$$

What is the equation of the asymptote of $y$ ?
A. $y=2$
B. $y=2 x$
C. $y=2 x+6$
D. $y=6$
E. $y=6 x+2$

7 (4 points) What types of asymptote(s) does the following function have

$$
f(x)=\frac{x^{2}-1}{(x-1)(2 x-3)}
$$

A. Horizontal asymptote $y=\frac{1}{2}$ and vertical asymptote $x=\frac{3}{2}$
B. Horizontal asymptote $y=1$
C. Vertical asymptote $x=1$
D. Horizontal asymptote $y=\frac{3}{2}$
E. No asymptote

8 (4 points) Let $y=3 x^{4}-4 x^{3}$. On the interval $[0,2]$, the minimum value of $y$ is obtained at $x=$ ?
A. 0
B. $1 / 4$
C. $1 / 2$
D. 1
E. 2

9 (4 points) Suppose that $f(x)$ is differentiable and $f^{\prime}(1)=0$. Which of the following statement is necessarily true?
A. $f(x)$ has relative max. at $x=1$
B. $f(x)$ has relative min. at $x=1$
C. $f(x)$ has no relative max. nor relative min. at $x=1$
D. $f(x)$ has critical point $(1, f(1))$
E. $f(x)$ has a point of inflection at $x=1$

10 (4 points) Find

$$
\int 1+3 x^{2} d x
$$

explicitly.
A. $6 x+C$
B. $1+6 x+C$
C. $x+6 x^{3}+C$
D. $x+3 x^{3}+C$
E. $x+x^{3}+C$

## Part 2: Long Answers (60 marks)

## Show your work for full marks

2. ( 15 points) The equation $e^{3 x}+e^{2 y}=10 x-4 y+2$ defines $y$ implicitly in terms of $x$ near the point $(0,0)$. Find an expression for $y^{\prime}$ in terms of $x$ and $y$ and evaluate this expression at $(0,0)$.
3. (15 points) Use Newton's method to approximate the solution to $f(x)=x^{4}-x-3=0$ between $x=1$ and $x=2$. Start at the point $x_{0}=1.5$ and perform two iterations. Be sure to leave at least 4 decimal places in your final answers.
4. Sketch the following curve

$$
f(x)=x^{3}-3 x^{2}+2
$$

by following the steps below.
(a) (3 points) Given that $f(1)=0$, find $x / y$ intercepts.
(b) (5 points) intervals on which $f$ increases/decreases and find relative max. and relative min.
(c) (5 points) intervals of concavity and point(s) of inflection.
(d) (2 points) Use the previous 3 parts to draw the curve. Be sure to label the $x / y$-intercepts, critical point(s), and point(s) of inflection.
5. Compute the following integrals
(a) (7 points)

$$
\int \frac{x}{x+1} d x
$$

(Hint: $x=x+1-1$.)
(b) (8 points) Suppose that

$$
y=\int 4 x^{3}+2 e^{x-1} d x
$$

and $y(1)=2$. Compute $y$ explicitly in terms of $x$.

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Multiple Choice Answer Sheet

| Question Number | Answer |
| :---: | :---: |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| 7 |  |
| 8 |  |
| 9 |  |
| 10 |  |
| 6 |  |
| 2 |  |

