Faculty of Arts and Science
University of Toronto

MAT133Y Term Test 1

Thursday May 31, 2018, 7:10 pm – 9:00 pm
Duration - 110 minutes

Surname: Solution

Given Name: ______________________

Student Number: ______________________

Tutorial Section: ______________________

Allowed Aid: A TI-30X IIS calculator, to be supplied by the student. **No other aid is allowed.**

Instruction:

- Fill in all required information on this cover sheet and **the multiple choice answer sheet on the last page of your exam. DO NO TEAR THE ANSWER SHEET. MARK WILL ONLY BE AWARDED TO ANSWERS ON THE ANSWER SHEET!**
- This exam contains 13 pages (including this cover page) and 5 problems. Once the exam begins, check to see if any pages are missing.
- Unless otherwise indicated, you are required to show your work on each problem on this exam. If you need more space, use the back of the pages; clearly indicate when you have done this.
- On the written portion of the test, **BLANK ANSWER WILL RECEIVE 2 MARKS PER QUESTION or 1 MARK PER SUBQUESTION.** Leave a question or a subquestion blank if you do not know how to do it. You will not be entitled to the free mark if you attempted the question.
- Questions are not order in increasing order of difficulties. Be sure to read through all the problems and plan your time well.

Section	*Time*	*Location*	*Instructor*
TUT0101	T 14 – 15	BA 2139	Tristan Milne
TUT0101	R 14 – 15	BA 2139	
TUT0102	T 14 – 15	BA 1230	Kai Wang
TUT0102	R 14 – 15	BA 1230	
TUT0103	T 14 – 15	BA 2195	Dmitri Chouchkov
TUT0103	R 14 – 15	BA 2195	
TUT0201	T 15 – 16	BA 2139	Tristan Milne
TUT0201	R 15 – 16	BA 2139	
TUT0202	T 15 – 16	BA 1230	Kai Wang
TUT0202	R 15 – 16	BA 1230	
TUT0203	T 15 – 16	BA 2195	Dmitri Chouchkov
TUT0203	R 15 – 16	BA 2195	

Problem	Points	Score
1	40	
2	15	
3	15	
4	15	
5	15	
Total:	100	

Part 1: Multiply Choice (40 marks)

Multiple Choice Questions

1. Multiple choice.

1 (4 points) An annual interest rate of 8% compounded semiannually corresponds to an annual effective rate of

A. 8%
B. 8.2031%
C. 12%
D. 9.2456%
(E.) 8.1600%

$r_e = \left(1+\frac{0.08}{2}\right)^2 - 1$

$= 0.0816$

$= 8.16\%$

2 (4 points) Determine the present value of $4000 due in 5 years if the interest rate is 10% compounded semiannually.

A. $2483.69
B. $1542.17
(C.) $2455.65
D. $3134.10
E. $2431.15

$PV = 4000\left(1+\frac{0.10}{2}\right)^{-10}$

$= \$2455.65$

3 (4 points) Suppose that a person invests now \$20,000 in a business venture that guarantees the same cash flow at the end of every quarter for 4 years. If the investment has a yield rate of 16% compounded quarterly, then each cash flow is

A. \$916.40
B. \$1527.52
C. \$1716.40
D. \$1917.39
E. \$2341.23

$A = R a_{\overline{n}|r}$

$20{,}000 = R a_{\overline{16}|0.04}$

$R = \frac{2000}{1 - 1.04^{-16}} \cdot 0.04$

$= \$1716.40$

4 (4 points) You bought \$100 worth of Stock B on Jan 1, 2015. On Jan 1, 2019, it is worth \$200 dollars. The effective annual rate of this investment over that period of time is closest to

A. 2.00%
B. 25.00%
C. 14.87%
D. 14.21%
E. 18.92%

$200 = 100(1+r)^4$

$r = \left(\frac{200}{100}\right)^{1/4} - 1$

$= 0.189207\ldots$

$= 18.92\%$

5 (4 points) If a \$700,000 mortgage, amortized over 25 years at 6% compounded semi-annually, has monthly payments, then each payment is closest to

A. \$4534.31
(B) \$4478.65
C. \$4468.21
D. \$2199.39
E. \$4211.23

$$\left(1+\frac{0.06}{2}\right)^2 = (1+i)^{12}$$

$$(1+i) = (1.03)^{1/6} \Rightarrow i = 0.00493862\ldots$$

$$A = R\, a_{\overline{n}|r}$$

$$700{,}000 = R\, a_{\overline{25\cdot 12}|i}$$

$$R = \frac{700{,}000}{1-(1+i)^{-25\cdot 12}}\cdot i = \frac{700{,}000}{1-1.03^{-50}}\cdot i$$

$$= \$4778.65$$

(\$4778.64 also acceptable, depends on rounding)

6 (4 points) A bond of face value \$100 pays coupon semi-annually. The price on September 1, 2000 of this bond with following rates is closest to

Issuer	Coupon Rate	Maturity Date	Yield to Maturity
Ontario	10.5%	March 1, 2029	5.9%

A. \$133.19
B. \$148.14
C. \$155.62
D. \$162.75
(E) \$163.10

coupon per $\frac{1}{2}$ yr =

$$100\cdot\frac{1}{2}\cdot 0.105 = \$5.25$$

$$PV = 100\left(1+\frac{0.059}{2}\right)^{-57} + 5.25\, a_{\overline{57}|\frac{0.059}{2}}$$

(Note: Sept 2000 to March 1, 2029 is $2\cdot 29 - 1$ months = 57 months)

$$= \$163.10$$

7 (4 points) If

$$\begin{pmatrix} 3x & -y \\ x & y \end{pmatrix} = \begin{pmatrix} 4x & 2 \\ 2x & y \end{pmatrix},$$

then $y =$

A. 4
B. 2
C. -4
D. 0
E. -2

$-y = 2$

$y = -2$

8 (4 points) The follow product of matrices is equal to

$$\begin{pmatrix} 1 & -1 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 0 & 2 \\ 5 & -3 \end{pmatrix},$$

A. $\begin{pmatrix} -2 & 8 \\ 3 & 8 \end{pmatrix}$

B. $\begin{pmatrix} 6 & 8 \\ -4 & -17 \end{pmatrix}$

C. $\begin{pmatrix} 4 & -1 \\ 0 & 6 \end{pmatrix}$

D. $\begin{pmatrix} -5 & 5 \\ 20 & -6 \end{pmatrix}$

E. $\begin{pmatrix} 0 & -2 \\ 15 & -12 \end{pmatrix}$

$= \begin{pmatrix} 1\cdot 0 + (-1)5 & 1\cdot 2 + (-1)(-3) \\ 3\cdot 0 + 4\cdot 5 & 3\cdot 2 + 4(-3) \end{pmatrix}$

9 (4 points) $\begin{pmatrix} 0 & 2 \end{pmatrix} \left[\begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix} - 5 \begin{pmatrix} 0 & 1 \\ 2 & 2 \end{pmatrix}^T \right] \begin{pmatrix} 0 \\ 1 \end{pmatrix} =$

A. $[-16]$
B. $[-12]$
C. $[-8]$
D. $[-4]$
E. $[0]$

$= (0, 2)\left(\begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix} - 5\begin{pmatrix} 0 & 2 \\ 1 & 2 \end{pmatrix}\right)\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$= (0, 2)\left(\begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix} - \begin{pmatrix} 0 & 10 \\ 5 & 10 \end{pmatrix}\right)\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$= (0, 2)\begin{pmatrix} 1 & -9 \\ -5 & -8 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$= (-16)$

10 (4 points) Given that $\begin{pmatrix} -4 & 2 \\ 3 & -1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$, the value of x in the following system of equations is

$$\begin{cases} x + 2y = 4 \\ 3x + 4y = -2 \end{cases}$$

A. 7
B. -8
C. 9
D. -10
E. 11

Eqn can be written as

$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$

Then multiply both sides by $\begin{pmatrix} -4 & 2 \\ 3 & -1 \end{pmatrix}$,

$\begin{pmatrix} -4 & 2 \\ 3 & -1 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -4 & 2 \\ 3 & -1 \end{pmatrix}\begin{pmatrix} 4 \\ -2 \end{pmatrix}$

$\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -16-4 \\ 12+2 \end{pmatrix}$

$2\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -20 \\ 14 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -10 \\ 7 \end{pmatrix}$

Part 2: Long Answers (60 marks)

Show your work for full marks

2. (a) (8 points) Suppose that you can invest \$5000 in a business that guarantees you the following cash flow: \$3000 at the end of 2 years, \$2000 at the end of 4 years, and \$2000 at the end of 6 years. Assume an interest rate of 6% compounded monthly, find the *net* present value of the cash flows. Is the investment profitable?

(1) (1)

$$NPV = 3000\left(1+\frac{0.06}{12}\right)^{-2\cdot 12} + 2000\left(1+\frac{0.06}{12}\right)^{-4\cdot 12} + 2000\left(1+\frac{0.06}{12}\right)^{-6\cdot 12} - 5000$$

(1) (1)

(2) $= \$632.36$

(2) Yes, profitable

(b) (7 points) Is the investment still profitable if the interest is 10% compounded continuously?

(1) (1)

$$NPV = 3000e^{-0.1\cdot 2} + 2000e^{-0.1\cdot 4} + 2000e^{-0.1\cdot 6} - 5000$$

(1) (1)

(2) $= \$-105.54$

(1) No, not profitable.

3. On Jan 1, 2010 a retiree had 2 annuity dues:

   (i) \$5000 payable on Jan 1 of each year and a final payment on Jan 1 of 2025
   (ii) \$800 payable on the beginning of each month with a final payment on Jan 1, 2025

   Immediately after receiving the first payment on Jan 1, 2010, he requested these 2 annuities be combined into a single annuity payable on Jan 1 and July 1 of each year with a final payment on Jan 1 2025. If all annuities/anuity dues are based on an **effective annual rate** of 6%, the find

   (a) (3 points) the rate per payment period of the three annuities.

yearly payment annuity (i) : $r = 0.06$

monthly (ii) : $r = (1+0.06)^{\frac{1}{12}} - 1$

$= 0.004867 = 0.4867\%$

semi-annual new annuity : $r = (1+0.06)^{1/2} - 1 = 2.9563\%$

   (b) (6 points) the value of each of the two old annuities immediately after the first payment on Jan 1, 2010.

Yearly (i). There are 15 payments after Jan 1, 2010 payment. Hence, present value A is

$A = 5000\, a_{\overline{15}|0.06} = \$48561.24(5)$

Monthly (ii). Total $15 \cdot 12 = 180$ payments after Jan 1, 2010 Present value A is

$A = 800\, a_{\overline{180}|0.004867} = \$95774.65(0)$

(c) (6 points) the value of the semiannual payment of the new annuity.

Total value of 2 old annuities after Jan 1, 2010 payment is

\$48561.24(5) + \$95774.65(0) = \$144335.89 (5)

Use $A = R a_{\overline{n}|i}$, get (3)

$144335.895 = R\, a_{\overline{30}|0.029563}$

↑ 2 · 12 yr

$\Rightarrow R = \$7322.37$ (3)

4. (15 points) For what values of a will the following system of equation have a solution? For each such a, find the solution.

$$\begin{cases} x - y - 3z = 2 \\ x + y - z = 1 \\ 2x - y - 5z = a \end{cases}$$

(5) Step

$$\left(\begin{array}{ccc|c} 1 & -1 & -3 & 2 \\ 1 & 1 & -1 & 1 \\ 2 & -1 & -5 & a \end{array}\right)$$

(5) Row reduction to obtain value of a

$$\xrightarrow{-R_1+R_2 \to R_2} \left(\begin{array}{ccc|c} 1 & -1 & -3 & 2 \\ 0 & 2 & 2 & -1 \\ 2 & -1 & -5 & a \end{array}\right) \xrightarrow{-2R_1+R_3 \to R_3} \left(\begin{array}{ccc|c} 1 & -1 & -3 & 2 \\ 0 & 2 & 2 & -1 \\ 0 & 1 & 1 & a-4 \end{array}\right)$$

$$\xrightarrow{\frac{1}{2}R_2 \to R_2} \left(\begin{array}{ccc|c} 1 & -1 & -3 & 2 \\ 0 & 1 & 1 & -1/2 \\ 0 & 1 & 1 & a-4 \end{array}\right) \xrightarrow{-1R_2+R_3 \to R_3} \left(\begin{array}{ccc|c} 1 & -1 & -3 & 2 \\ 0 & 1 & 1 & -1/2 \\ 0 & 0 & 0 & a-7/2 \end{array}\right)$$

$a = 7/2$ has solution

(5) Obtaining the soln. Since $a = 7/2$, have

$$\left(\begin{array}{ccc|c} 1 & -1 & -3 & 2 \\ 0 & 1 & 1 & -1/2 \\ 0 & 0 & 0 & 0 \end{array}\right) \xrightarrow{R_2+R_1 \to R_1} \left(\begin{array}{ccc|c} 1 & 0 & -2 & 3/2 \\ 0 & 1 & 1 & -1/2 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

Let $z = t$, the soln is

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3/2 + 2t \\ -1/2 - t \\ t \end{pmatrix}$$ for any # t.

5. (a) (10 points) Find the inverse of the following matrix

$$\begin{pmatrix} 0 & -3 & -2 \\ 1 & -4 & -2 \\ -3 & 4 & 1 \end{pmatrix}$$

(Hint: you should not see any fraction in your final answer)

(5) Step the problem in matrix row reduction augmented matrix form, and attempt to reduce

$$\left(\begin{array}{ccc|ccc} 0 & -3 & -2 & 1 & 0 & 0 \\ 1 & -4 & -2 & 0 & 1 & 0 \\ -3 & 4 & 1 & 0 & 0 & 1 \end{array}\right) \xrightarrow{R_2 \leftrightarrow R_1} \left(\begin{array}{ccc|ccc} 1 & -4 & -2 & 0 & 1 & 0 \\ 0 & -3 & -2 & 1 & 0 & 0 \\ -3 & 4 & 1 & 0 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{3R_1+R_3 \to R_3} \left(\begin{array}{ccc|ccc} 1 & -4 & -2 & 0 & 1 & 0 \\ 0 & -3 & -2 & 1 & 0 & 0 \\ 0 & -8 & -5 & 0 & 3 & 1 \end{array}\right) \xrightarrow{-\frac{1}{3}R_2 \to R_2} \left(\begin{array}{ccc|ccc} 1 & -4 & -2 & 0 & 1 & 0 \\ 0 & 1 & 2/3 & -1/3 & 0 & 0 \\ 0 & -8 & -5 & 0 & 3 & 1 \end{array}\right)$$

$$\xrightarrow{8R_2+R_3 \to R_3} \left(\begin{array}{ccc|ccc} 1 & -4 & -2 & 0 & 1 & 0 \\ 0 & 1 & 2/3 & -1/3 & 0 & 0 \\ 0 & 0 & 1/3 & -\frac{8}{3} & 3 & 1 \end{array}\right) \xrightarrow{3R_3 \to R_3} \left(\begin{array}{ccc|ccc} 1 & -4 & -2 & 0 & 1 & 0 \\ 0 & 1 & 2/3 & -1/3 & 0 & 0 \\ 0 & 0 & 1 & -8 & 9 & 3 \end{array}\right)$$

$$\xrightarrow{-\frac{2}{3}R_3+R_2 \to R_2} \left(\begin{array}{ccc|ccc} 1 & -4 & -2 & 0 & 1 & 0 \\ 0 & 1 & 0 & 5 & -6 & -2 \\ 0 & 0 & 1 & -8 & 9 & 3 \end{array}\right) \xrightarrow{2R_3+R_1 \to R_1} \left(\begin{array}{ccc|ccc} 1 & -4 & 2 & -16 & 19 & 6 \\ 0 & 1 & 0 & 5 & -6 & -2 \\ 0 & 0 & 1 & -8 & 9 & 3 \end{array}\right)$$

$$\xrightarrow{4R_2+R_1 \to R_1} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 4 & -5 & -2 \\ 0 & 1 & 0 & 5 & -6 & -2 \\ 0 & 0 & 1 & -8 & 9 & 3 \end{array}\right)$$

(5)

$$\text{Inverse} = \begin{pmatrix} 4 & -5 & -2 \\ 5 & -6 & -2 \\ -8 & 9 & 3 \end{pmatrix}$$

(b) (5 points) Use part (a) to solve the following system of equations.

$$\begin{cases} -3y - 2z = 1 \\ x - 4y - 2z = 1 \\ -3x + 4y + z = 2 \end{cases}$$

Write the equation as

$$\underbrace{\begin{pmatrix} 0 & -3 & -2 \\ 1 & -4 & -2 \\ -3 & 4 & 1 \end{pmatrix}}_{A} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$$

we computed A^{-1} in part a). Thus

$$A^{-1}A\begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^{-1}\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 & -5 & -2 \\ 5 & -6 & -2 \\ -8 & 9 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$$

(5)

−1 or −2 for calculation errors

$$= \begin{pmatrix} 4 - 5 - 4 \\ 5 - 6 - 4 \\ -8 + 9 + 6 \end{pmatrix} = \begin{pmatrix} -5 \\ -5 \\ 7 \end{pmatrix}$$

(2) only if you didn't use part a).

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Thursday May 31, 2018, 7:10 pm – 9:00 pm
Duration - 110 minutes

Surname: ______________________________

Given Name: ______________________________

Student Number: ______________________________

Tutorial Section: ______________________________

Multiple Choice Answer Sheet

Question Number	Answer
1	E
2	C
3	C
4	E
5	B
6	E
7	E
8	D
9	A
10	D