Solns

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Department of Mathematics
University of Toronto

Tuesday, Oct. 31, 2017, 6:10-8:00 PM

MAT 133Y TERM TEST #1

Calculus and Linear Algebra for Commerce

Duration: 1 hour 50 minutes

Aids Allowed: A TI-30X IIS calculator, to be supplied by student. **No other calculator is permitted.**

Instructions: Fill in the information on this page, and make sure your test booklet contains 11 pages. In addition, you should have a **multiple-choice answer sheet**, on which you should fill in your name, number, tutorial time, tutorial room, and tutor's name.

This test consists of 10 multiple choice questions, and 4 written-answer questions.
For the **multiple choice questions** you can do your rough work in the test booklet, but you must record your answer by circling the appropriate letter **on the answer sheet** with your pencil. Each correct answer is worth 4 marks; a question left blank, or an incorrect answer, or two answers for the same question is worth 0. For the **written-answer questions**, present your solutions in the space provided. The value of each written-answer question is indicated beside it.

ENCLOSE YOUR FINAL ANSWER IN A BOX AND WRITE IT IN INK.

TOTAL MARKS: 100

FAMILY NAME: ______________________

GIVEN NAME: ______________________

STUDENT NO: ______________________

SIGNATURE: ______________________

TUTORIAL TIME and ROOM: ______________________

REGCODE and TIMECODE: ______________________

T.A.'S NAME: ______________________

Regcode	Timecode	Room	**Regcode**	Timecode	Room
T0101	M9A	RS310	T0502	W3B	GB120
T0102	M9B	BA2135	T0503	W3C	UC65
T0103	M9C	HA316	T0601	R4A	MP137
T0104	M9D	LM157	T0602	R4B	SS2106
T0201	M3A	ES4001	T0603	R4C	FG139
T0202	M3B	UC87	T0604	R4D	SS2105
T0203	M3C	BA3012	T0701	F2A	BA2135
T0204	M3D	SS2127	T0702	F2B	SS2105
T0301	T3A	MS3278	T0703	F2C	BA2165
T0302	T3B	UC144	T0801	F3A	ES4000
T0303	T3C	BA1220	T0802	F3B	SS2105
T0304	T3D	BA3012	T0803	F3C	RW143
T0401	W9A	AB107	T5101	M5A	MS4171
T0402	W9B	BA2185	T5102	M5B	BA2175
T0403	W9C	LM157	T5103	M5C	BA2139
T0501	W3A	BA3116			

FOR MARKER ONLY	
Multiple Choice	
B1	
B2	
B3	
B4	
TOTAL	

A

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PART A. Multiple Choice

1. *[4 marks]*

If $A = \begin{pmatrix} 0 & 2 & 1 \\ 2 & 0 & 0 \\ x & 0 & 4 \end{pmatrix}$, and $A = A^T$, then $x =$

$$A^T = \begin{pmatrix} 0 & 2 & x \\ 2 & 0 & 0 \\ 1 & 0 & 4 \end{pmatrix} = \begin{pmatrix} 0 & 2 & 1 \\ 2 & 0 & 0 \\ x & 0 & 4 \end{pmatrix} = A$$

so $x = 1$ B

or If $A = (a_{ij})$, $A^T = (a_{ji})$

so $a_{13} = a_{31}$

$x = 1$ B

A. 0

B. 1

C. 2

D. 4

E. 7

2. *[4 marks]*

Find $\begin{pmatrix} 2 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 2 \\ 3 \end{pmatrix}$. $= \begin{pmatrix} 2 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 2 \\ 3 \end{pmatrix}$ since the 4×4 matrix is I.

$$= 2 \cdot 0 + 0 \cdot 1 + 1 \cdot 2 + 1 \cdot 3$$
$$= 5 \quad \text{B}$$

A. (2)

B. (5)

C. (4)

D. (1)

E. (8)

A

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3. *[4 marks]*

Find $\begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix}^4$.

$$\begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix}^2 = \begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 4 & 3 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix}^4 = \left[\begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix}^2\right]^2 = \begin{pmatrix} 4 & 3 \\ 0 & 1 \end{pmatrix}^2$$

$$= \begin{pmatrix} 4 & 3 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 4 & 3 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 16 & 15 \\ 0 & 1 \end{pmatrix} \quad \text{A}$$

A. $\begin{pmatrix} 16 & 15 \\ 0 & 1 \end{pmatrix}$

B. $\begin{pmatrix} 12 & 7 \\ 1 & 0 \end{pmatrix}$

C. $\begin{pmatrix} 16 & 1 \\ 0 & 1 \end{pmatrix}$

D. $\begin{pmatrix} 10 & 4 \\ 8 & 0 \end{pmatrix}$

E. $\begin{pmatrix} 18 & 4 \\ 11 & 2 \end{pmatrix}$

4. *[4 marks]*

Find x and y if $\begin{pmatrix} x & 0 \\ y & x \end{pmatrix}\begin{pmatrix} 0 & 1 \\ x & y \end{pmatrix} = \begin{pmatrix} 0 & -2 \\ 4 & 7 \end{pmatrix}$.

$$\begin{pmatrix} x & 0 \\ y & x \end{pmatrix}\begin{pmatrix} 0 & 1 \\ x & y \end{pmatrix} = \begin{pmatrix} 0 & x \\ x^2 & y+xy \end{pmatrix} = \begin{pmatrix} 0 & -2 \\ 4 & 7 \end{pmatrix}$$

So $x = -2$ $\quad x^2 = 4$

$y + xy = 7$

$-y = 7$

$x = -2$

$y = -7$ $\quad$ E

A. $x = 3$, $y = 7$

B. $x = -2$, $y = 5$

C. $x = 4$, $y = -1$

D. $x = -2$, $y = -2$

E. $x = -2$, $y = -7$

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5. *[4 marks]*

The system

$$\begin{array}{rcrcrcr} x & + & 2y & - & z & = & -1 \\ 2x & - & y & + & 3z & = & 8 \\ 5x & & & + & 5z & = & 15 \end{array}$$

has:

$$\left(\begin{array}{ccc|c} 1 & 2 & -1 & -1 \\ 2 & -1 & 3 & 8 \\ 5 & 0 & 5 & 15 \end{array}\right) \xrightarrow[R_3 \to -5R_1+R_3]{R_2 \to -2R_1+R_2} \left(\begin{array}{ccc|c} 1 & 2 & -1 & -1 \\ 0 & -5 & 5 & 10 \\ 0 & -10 & 10 & 20 \end{array}\right)$$

$$\xrightarrow[R_3 \to 10R_2+R_3]{R_2 \to -\frac{1}{5}R_2} \left(\begin{array}{ccc|c} 1 & 2 & -1 & -1 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow \text{2 equations, 3 unknowns so 1 parameter}$$

$$\xrightarrow{R_1 \to -2R_2+R_1} \left(\begin{array}{ccc|c} 1 & 0 & 1 & 3 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$x = 3 - z$
$y = -2 + z$

D

A. a unique solution with $x = 1$, $y = 0$, $z = 2$.

B. a unique solution with $x = -1$, $y = 2$, $z = 4$.

C. a 2-parameter family of solutions.

D. a 1-parameter family of solutions with $x = 3 - z$, $y = -2 + z$, $z \in \mathbb{R}$.

E. a 1-parameter family of solutions with $x = 1 - y$, $z = -2 + y$, $y \in \mathbb{R}$.

Note that E does not satisfy the 3rd equation.

6. *[4 marks]*

After five years in a bank account, an initial deposit of \$1,000.00 yields an account balance of \$1,007.52. The account accrued at an effective annual rate closest to:

A. 7.52%

B. 1.51%

C. 0.75%

D. 0.32%

E. 0.15%

Let r = effective annual rate.

$1007.52 = 1000(1+r)^5$

$r = (1.00752)^{\frac{1}{5}} - 1$

$\approx .001499$

$.15\%$ is closest

E

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7. *[4 marks]*

Six semi-annual deposits of $1,000 are made into an account that earns 2% per year compounded semi-annually. Immediately after the last deposit, the account balance is closest to:

$r = .01 \quad n = 6$

A. $5,795.48

$S = 1000\, s_{\overline{6}|.01}$

B. $5,853.43

$= 1000 \dfrac{[(1.01)^6 - 1]}{.06}$

C. $6,152.02

D. $6,213.54

≈ 6152.02 C

E. $6,308.12

8. *[4 marks]*

A $500 bond with semi-annual coupon payments (the next one in 6 months) and an annual coupon rate of 5% matures in 15 years. If the annual yield rate is 7%, then the price of the bond is closest to:

A. $408.92

$V = 500 \quad n = 30$

B. $604.05

$r = .025 \quad i = .035$

C. $408.04

$P = 500(1.035)^{-30} + .025 \times 500\, a_{\overline{30}|.035}$

D. $432.24

≈ 408.04 C

E. $603.80

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9. *[4 marks]*

A bond with 24 semi-annual coupon payments remaining (the next one in 6 months) and an annual coupon rate of 6% sells for \$272.36. If the annual yield to maturity is 5%, then the face value of the bond is closest to:

A. \$200

B. \$225

C. \$250

D. \$275

E. \$300

$n = 24 \quad r = .03 \quad P = 272.36 \quad i = .025$

$272.36 = V(1.025)^{-24} + .03V\, a_{\overline{24|}.025}$

$$\frac{272.36}{(1.025)^{-24} + .03\, a_{\overline{24|}.025}} = V$$

$V = 250.003$ C

10. *[4 marks]*

If a loan is amortized over n periods with fixed payments of R at the end of each period at a fixed rate of r per period, then the principal repaid in the last payment is

A. R

B. $R - rR$

C. rR

D. $R(1+r)^{-1}$

E. $\frac{R}{r}$

Since there is 1 paymt. remaining, the principal outstanding at the beginning of the last period is $R(1+r)^{-1}$.

Method 1: In the next (which is the last) paymt., the interest for this period is paid, and then the remaining principal outstanding which is $R(1+r)^{-1}$ as already found. D

Method 2: The interest in the last paymt is $r \times P.O = rR(1+r)^{-1}$. The principal in this paymt is $R - rR(1+r)^{-1} = R\left[1 - \frac{r}{1+r}\right] = R\left[\frac{1+r-r}{1+r}\right]$

$= \frac{R}{1+r}$ D

A

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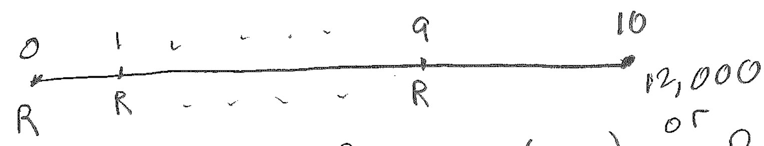
PART B. Written-Answer Questions

1. *[16 marks]*

[6] (a) At the end of 10 years, a company wants to replace equipment that will then be worth \$12,000. To do this, they will make ten equal annual payments starting now into an account that earns 3% per annum. Find the amount of each payment.



$$12{,}000 = R\,s_{\overline{10}|.03} \times (1.03) \overset{\text{or}}{=} R\,s_{\overline{11}|.03} - R$$

$$R = \frac{12{,}000}{1.03\,s_{\overline{10}|.03}} \overset{\text{or}}{=} \frac{1200}{[s_{\overline{11}|.03} - 1]}$$

$$= \boxed{\$1016.28}$$

(b) *[5]* (i) A mortgage for \$200,000 is amortized over 20 years at a semi-annual rate of 3% with monthly payments. Find the amount of each monthly payment.

$$200{,}000 = R\,a_{\overline{240}|i} \qquad R = \frac{200{,}000}{a_{\overline{240}|i}} = \frac{200{,}000\,i}{1-(1+i)^{-240}}$$

with $(1+i)^{12} = (1.015)^2$

$$R = 200{,}000\,\frac{[(1.015)^{\frac{1}{6}} - 1]}{1-(1.015)^{-40}}$$

$$= \boxed{\$1107.34}$$

Unnecessary, but usable $i = .002484517$

with $(1+i)^{12} = (1.03)^2$

$$R = 200{,}000\,\frac{[(1.03)^{\frac{1}{6}} - 1]}{1-(1.03)^{-40}}$$

$$= \boxed{\$1424.38}$$

Unnecessary, but usable $i = .004938622$

[5] (ii) Find the principal outstanding at the beginning of the 10^{th} month.

The beginning of the 10th month is just after the 9th payment, so 231 remain.

$$P.O. = R\,a_{\overline{231}|i} = \frac{200{,}000}{a_{\overline{240}|i}}\,a_{\overline{231}|i} = 200{,}000\,\frac{[1-(1+i)^{-231}]}{[1-(1+i)^{-40}]}$$

with $(1+i)^{12} = (1.015)^2$

$(1+i)^{231} = (1.015)^{\frac{231}{6}}$

$$P.O. = 200{,}000\,\frac{[1-(1.015)^{-\frac{231}{6}}]}{[1-(1.015)^{-40}]}$$

$$= \boxed{\$194{,}451.19}$$

with $(1+i)^{12} = (1.03)^2$

$(1+i)^{231} = (1.03)^{231/6}$

$$P.O. = 200{,}000\,\frac{[1-(1.03)^{-\frac{231}{6}}]}{[1-(1.03)^{-40}]}$$

$$= \boxed{\$195{,}991.59}$$

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2. *[15 marks]*

(a) A loan of \$100,000 at 10% per year compounded semi-annually with payments at the end of each 6 month period has regular payments of \$15,000 each, except for the last one which is smaller.

[5] (i) How many payments of \$15,000 are there?

$$100{,}000 = 15{,}000\, a_{\overline{n}|.05} = 15{,}000\left[\frac{1-(1.05)^{-n}}{.05}\right]$$

$$100{,}000 = 300{,}000\left[1-(1.05)^{-n}\right]$$

$$\frac{1}{3} = 1-(1.05)^{-n}$$

$$(1.05)^{-n} = \frac{2}{3}$$

$$(1.05)^{n} = \frac{3}{2} = 1.5$$

$$n \ln 1.05 = \ln 1.5$$

$$n = \frac{\ln 1.5}{\ln 1.05} \approx 8.31$$

There are $\boxed{\text{8 full payments}}$

[4] (ii) How big is the last (smaller) payment?

$$\begin{array}{ccccc} 0 & 1 & \cdots & 8 & 9 \\ 100{,}000 & 15{,}000 & \cdots & 15{,}000 & X \end{array}$$

$$100{,}000 = 15{,}000\, a_{\overline{8}|.05} + X(1.05)^{-9} \quad \text{or} \quad 100{,}000(1.05)^{9} = 15{,}000\, s_{\overline{8}|.05}(1.05) + X$$

From either one, $\boxed{X = \$4734.36}$

Notice that $.31 \times 15{,}000 = \$4650$ (and even using $.31038 6223$ gives only \$4655.79)

which is quite far off.

[6] (b) A sum of \$100,000 was invested on Sept. 1, 2000 at a rate of 5% compounded annually in order to provide an annual scholarship of \$20,000 every Sept. 1, forever, starting as soon as possible. Assuming that interest is paid every August 31, in what year will the first payment of \$20,000 be made?

Sept 1, 2000

$$\begin{array}{cccc} 0 & 1 & \cdots & n \\ 100{,}000 & & & A \end{array}$$

$$100{,}000(1.05)^{n} = A = \frac{R}{r} = \frac{20{,}000}{.05} = 400{,}000$$

$$(1.05)^{n} = 4 \qquad n = \frac{\ln 4}{\ln 1.05} \approx 28.413$$

$$n \ln 1.05 = \ln 4$$

In 2028 there won't be enough money, but by 2029 there will be enough and a little over. The first payment will be a yr. after that in 2030.

$$\begin{array}{ccc} 0 & 1 & 2028 \\ 100{,}000 & \longrightarrow & 100{,}000(1.05)^{28} \end{array}$$

A

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3. *[13 marks]*

A dietitian wants to combine three foods so that the resulting mixture contains 900 units of vitamins, 750 units of minerals, and 350 units of fats. The units of vitamins, minerals, and fats contained in each gram of the three foods are shown below. How many grams of each food should be combined to obtain the required mixture?
[*Make sure to show your system of equations and how you solve them. A correct answer by itself will get zero.*]

	Vitamins	Minerals	Fats
1 gram of food A	35 units	15 units	10 units
1 gram of food B	10 units	20 units	10 units
1 gram of food C	20 units	15 units	5 units

Let X_A, X_B, X_C be the no. of gms of foods A, B, C resp.

$$\left.\begin{aligned} 35X_A + 10X_B + 20X_C &= 900 \\ 15X_A + 20X_B + 15X_C &= 750 \\ 10X_A + 10X_B + 5X_C &= 350 \end{aligned}\right\}$$

$$\left(\begin{array}{ccc|c} 35 & 10 & 20 & 900 \\ 15 & 20 & 15 & 750 \\ 10 & 10 & 5 & 350 \end{array}\right)$$

$$\xrightarrow[R_3 \to \frac{1}{5}R_3]{R_1 \to \frac{1}{5}R_1,\ R_2 \to \frac{1}{5}R_2} \left(\begin{array}{ccc|c} 7 & 2 & 4 & 180 \\ 3 & 4 & 3 & 150 \\ 2 & 2 & 1 & 70 \end{array}\right)$$

$$\xrightarrow[R_1 \to \frac{1}{2}R_1]{R_1 \leftrightarrow R_3} \left(\begin{array}{ccc|c} 1 & 1 & \frac{1}{2} & 35 \\ 3 & 4 & 3 & 150 \\ 7 & 2 & 4 & 180 \end{array}\right) \xrightarrow[R_3 \to -7R_1 + R_3]{R_2 \to -3R_1 + R_2} \left(\begin{array}{ccc|c} 1 & 1 & \frac{1}{2} & 35 \\ 0 & 1 & \frac{3}{2} & 45 \\ 0 & -5 & \frac{1}{2} & -65 \end{array}\right)$$

$$\xrightarrow{R_3 \to 5R_2 + R_3} \left(\begin{array}{ccc|c} 1 & 1 & \frac{1}{2} & 35 \\ 0 & 1 & 3/2 & 45 \\ 0 & 0 & 8 & 160 \end{array}\right) \xrightarrow{R_3 \to \frac{1}{8}R_3} \left(\begin{array}{ccc|c} 1 & 1 & \frac{1}{2} & 35 \\ 0 & 1 & 3/2 & 45 \\ 0 & 0 & 1 & 20 \end{array}\right)$$

Back substitution says $\boxed{X_C = 20}$

$\boxed{X_B = 45 - \frac{3}{2}X_C = 15}$

$\boxed{X_A = 35 - \frac{1}{2}X_C - X_B = 35 - 10 - 15 = 10}$

or complete reduction:

$$\xrightarrow[R_1 \to -R_2 - \frac{1}{2}R_3 + R_1]{R_2 \to -\frac{3}{2}R_3 + R_2} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 10 \\ 0 & 1 & 0 & 15 \\ 0 & 0 & 1 & 20 \end{array}\right)$$

giving $\boxed{\begin{aligned} X_A &= 10 \\ X_B &= 15 \\ X_C &= 20 \end{aligned}}$ as before

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4. *[16 marks]*

Let $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{pmatrix}$, $C = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$.

[4] (a) Find A^{-1} or show A^{-1} does not exist.

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 2 & 0 & 0 & 1 \end{array}\right) \xrightarrow{R_3 \to -R_1 + R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{R_3 \to -R_2 + R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & -1 & 1 \end{array}\right)$$

$\boxed{A^{-1} \text{ does not exist}}$

[4] (b) Find B^{-1} or show B^{-1} does not exist.

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 3 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 2 & 0 & 0 & 1 \end{array}\right) \xrightarrow{R_3 \to -R_1 + R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 3 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{R_3 \to -R_2 + R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 3 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & -1 & -1 & 1 \end{array}\right)$$

$$\xrightarrow{R_3 \to -R_3}$$

$$R_1 \to -3R_3 + R_1 \quad \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & -3 & 3 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & -1 \end{array}\right)$$

$$B^{-1} = \begin{pmatrix} -2 & -3 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & -1 \end{pmatrix}$$

Question 4 continued on next page.

4. Continued...

[4] (c) How many solutions, if any, does $AX = C$ have?

$$\left(\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 2 & 2 \end{array}\right) \xrightarrow{R_3 \to -R_1 + R_3} \left(\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{R_3 \to -R_2 + R_3} \left(\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

2 eqns, 3 vars. $\Rightarrow$ 1 param.

$\boxed{\text{There are } \infty\text{-many solns}}$

[4] (d) How many solutions, if any, does $BX = C$ have? Find the solution(s).

Since B^{-1} exists

$X = \boxed{B^{-1}C \text{ is the unique soln}}$

$$X = \begin{pmatrix} -2 & -3 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} -2-3+6 \\ 0+1+0 \\ 1+1-2 \end{pmatrix}$$

$$\boxed{X = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}}$$