

University of Toronto
Faculty of Arts and Science
MAT136H1Y Midterm
Li Chen
Summer 2017
Duration - 110 minutes
No Aids Permitted

Surname: _____

Given Name: _____

Student Number: _____

This exam contains 10 pages (including this cover page) and 6 problems (one problem is bonus). Once the exam begins, check to see if any pages are missing and ensure that all required information at the top of this page has been filled in.

No aids are permitted on this examination. Examples of illegal aids include but are not limited to textbooks, notes, calculators, cellphones, or any electronic device.

Unless otherwise indicated, you are required to **show your work** on each problem on this exam. The following rules apply:

- **Total points available is 110; but the test is out of 100.** The last problem is a bonus problem.
- **Unreadable answers will be receive no mark.**
- **Organize your work** in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit.** A correct answer, unsupported by calculations, explanation, or algebraic work, will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- **If you need more space,** use the back of the pages; clearly indicate when you have done this.

Problem	Points	Score
1	40	
2	15	
3	15	
4	15	
5	15	
6	0	
Total:	100	

Part 1: Short Answers (50 marks)

No justification is necessary and no mark will be awarded for them

1. For each of the following questions, write your final answer in the box on the righthand side. Only your final answer will be graded.

(a) (5 points) True or False? Suppose that $f(x)$ is continuous on $[0, 1]$, then $f(x)$ is integrable on $[0, 1]$.

Final Answer

(b) (5 points) Given that $\frac{1}{3} = \int_0^{10} f(x)dx$, compute

$$\int_0^{10} 3f(x) + 2x dx$$

Final Answer

(c) (5 points) Compute $\frac{d}{dx} \left(x + \int_0^5 \sec^2(t) dt \right)$.

Final Answer

(d) (5 points) Find a value of x such that the function $f(x)$, given by

$$f(x) := \int_{-1}^x \frac{t^2}{1+t^4} dt,$$

reaches local maximum on $[0, \infty)$.

Final Answer

For the following problems, evaluate each definite integral.

(e) (5 points)

$$\int_0^{\ln(2)} \frac{2}{3} e^{2x+1} dx$$

Final Answer

(f) (5 points)

$$\int_0^{\pi} \sin(x) \cos(x) dx$$

Final Answer

(g) (5 points)

$$\int_{-\pi}^{\pi} \cos^2(x) \sin(x) dx$$

Final Answer

(h) (5 points)

$$\int_{\frac{\sqrt{2}}{2}}^1 \frac{1}{x^2 \sqrt{1-x^2}} dx$$

Final Answer

Part 1: Long Answers (50 marks)**Show your work for full marks**

2. (15 points)

$$\int \frac{1}{(x^2 + 2x + 2)(x^2 + 2x + 1)} dx$$

3. Compute the following integrals.

(a) (5 points)

$$\int \frac{1}{x^2 + 4x + 5} dx$$

(b) (5 points)

$$\int \frac{2x + 4}{x^2 + 4x + 5} dx$$

(c) (5 points) Use parts (a) and (b) to compute the following integral.

$$\int \frac{x + 3}{x^2 + 4x + 5} dx$$

4. (15 points)

$$\int \cos(x) \ln(\sin(x)) dx$$

5. (15 points) Let S be the region bounded by $x^2(y - 1) = 9$, $y = 0$, $x = 1$ and $x = 3$. Find the volume of the solid obtained by rotating S about the x -axis.

6. (Bonus: 10 points) Integrate

$$\int \frac{1}{x^{1/2} - x^{1/6}} dx$$

**This page is for additional work
and will not be marked.**

Formula Sheet

This page will not be marked.

Pythagorean	$\sin^2(x) + \cos^2(x) = 1$ $\tan^2(x) + 1 = \sec^2(x)$ $1 + \cot^2(x) = \csc^2(x)$
Double/Half Angle	$\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$ $\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$
Sum/Product	$\sin(x) \cos(y) = \frac{1}{2}(\sin(x - y) + \sin(x + y))$ $\sin(x) \sin(y) = \frac{1}{2}(\cos(x - y) - \cos(x + y))$ $\cos(x) \cos(y) = \frac{1}{2}(\cos(x - y) + \cos(x + y))$