University of Toronto
Faculty of Arts and Science

MAT136H1Y Final Exam

Instructor: Li Chen
August 2017
Duration - 180 minutes
No Aids Permitted

Surname: ______________________________

Given Name: ______________________________

Student Number: ______________________________

This exam contains 10 pages (including this cover page) and 5 problems (one problem is bonus). Once the exam begins, check to see if any pages are missing and ensure that all required information at the top of this page has been filled in.

No aids are permitted on this examination. Examples of illegal aids include but are not limited to textbooks, notes, calculators, cellphones, or any electronic device.

Unless otherwise indicated, you are required to **show your work** on each problem on this exam. The following rules apply:

- **Total points available is 110; but the test is out of 100.** The last problem is a bonus problem.
- **Unreadable answers will receive no mark.**
- **Organize your work** in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work, will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- **If you need more space**, use the back of the pages; clearly indicate when you have done this.

Problem	Points	Score
1	40	
2	15	
3	15	
4	15	
5	15	
Total:	100	

Part 1: Short Answers (40 marks)

No justification is necessary and no mark will be awarded for them

1. For each of the following questions, write your final answer in the box on the right hand side. Only your final answer will be graded.

   (a) (5 points) True of False? Let $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ are two sequences of real numbers. If $\sum_{n=1}^{\infty} a_n$ is divergent and $\sum_{n=1}^{\infty} b_n$ is convergent, then $\sum_{n=1}^{\infty} a_n + b_n$ convergent.

   Final Answer

   (b) (5 points) True or False? Suppose that we have a sequence real numbers $a_1, a_2, a_3, ...$ such that $a_n < 1/n^2$ if $n \geq 1000$. Then $\sum_{n=1}^{\infty}$ is convergent.

   Final Answer

   (c) (5 points) Assume that $\sum_{n=1}^{\infty} a_n = 2$. Compute $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} - a_n$

   Final Answer

   (d) (5 points) Let

   $$f(x) := e^{-x}$$

   What is the 100-th Taylor coefficient of the Taylor Series of $f(x)$ centered at $x = 0$?

   Final Answer

   All of the following series are convergent, compute the value of each series explicitly.

   (e) (5 points)

   $$\sum_{n=1}^{\infty} 2\frac{\cos(\pi + n\pi)}{9^n}$$

   Final Answer

(f) (5 points)

$$\sum_{n=1}^{\infty} \frac{1}{n^2 + 5n + 6}$$

Final Answer

(g) (5 points)

$$\sum_{n=0}^{\infty} \frac{(-1)^n \pi^n}{(2n+2)!}$$

Final Answer

(h) (5 points)

$$\sum_{n=0}^{\infty} \ln[\sec^2(n) - \tan^2(n)]$$

Final Answer

Part 1: Long Answers (60 marks)

Show your work for full marks

2. Determine if the following series are absolutely convergent, conditionally convergent, or divergent.

   (a) (5 points)

$$\sum_{n=2}^{\infty} \frac{n^n(-1)^n}{n!(n+1)}$$

(b) (10 points)

$$\sum_{n=3}^{\infty} \frac{(-1)^n}{10\ln(n) - 1/2^n}$$

3. (15 points) For each integer $k > 0$, find the radius of convergence of

$$\sum_{n=0}^{\infty} \frac{(n!)^k}{(kn)!} x^n$$

4. (a) (5 points) Find a closed form of

$$\sum_{n=0}^{\infty} \frac{x^n}{n}$$

(b) (5 points) Let $f(x) = \ln(1-x)/x$ if $x > 0$ and $f(0) = 1$ if $x = 0$. Show that f is continuous on $[0, 1)$

(c) (5 points) Assume that $\int_0^1 \frac{\ln(1-x)}{x} = -\pi^2/6$. Use parts (a) and (b) to compute

$$\sum_{n=1}^{\infty} \frac{1}{n^2}$$

5. (a) (8 points) Suppose that a sequence of real numbers, $\{a_n\}_{n=0}^{\infty}$, satisfies

$$a_{n+1} = -\frac{a_n}{(n+1)}$$

with $a_0 = 1$. Find a closed form formula for a_n in terms of n only. (Use the notation $n! = n(n-1)(n-2)\cdots(2)(1)$ and define $0! = 1$)

(b) (7 points) Solve the differential equation

$$f' = -f \text{ and } f(0) = 1$$

by writing f as a series representation $\sum_{n=0}^{\infty} a_n x^n$. Then determine the radius of convergence of this series.

This page is for additional work and will not be marked.