MAT 1347 HS
Instructor: B. Khesin

Graduate course
"Integrable Systems"
Spring 2020

Topics/problems for minipapers (3-4pp):

1. Derivation of the Camassa-Holm equation as the H^1-Euler equation on the group $\mathrm{Diff}(S^1)$.

2. The Camassa-Holm equation as the H^1-Euler equation on the Virasoro group.

3. Classify coadjoint orbits for the group $\mathrm{Diff}(S^1)$ (ref: Kirillov).

4. The bihamiltonian structure of the NLS.

5. Poisson properties of the Hasimito transformations from the binormal (filament) equation to the NLS one.

6. Comparison of solitons in the periodic and fast decaying cases of the KdV or NLS.

7. Symplectic structure on polygons as an approximation of the symplectic structure on arc-parameterized curves in $\mathbf{R}^3$.

8. Find the recurrence and explicit formulas for the KdV conservation laws.

9. First integrals of the KdV from the WKB asymptotics.

10. Angle-action variables for the KdV (ref: Kappeler et al)

11. An example of Arnold's diffusion (ref: Kaloshin-Levi)

12. Symplectic non-squeezing results for the KdV (ref: Kuksin)

13: The KAM approach to outer billiards (ref: Moser, Douady)

14. The relation between special solutions of the Calogero-Moser systems and the KdV/KP equations.

15. Describe the Landau-Lifschitz equation as a bihamiltonian system.

16. Explain the main features of the Ziglin method of proving non-integrability.

17. A Lax form for the pentagram maps (ref: Soloviev, Izosimov)

18. Continuous limits for pentagram maps.

19. The bicycle transformation on curves and its properties (ref: Tabachnikov)

20. Geometric discretization of the Toda system (ref: Doliwa)

21. Polygon recutting as an integrable system (ref: Adler)

22. Comparison of different definitions of discrete integrable systems.

(Topics outside of the list are also possible: please, discuss with the instructor.)