

**Exterior differential:**

$$\omega = e^{x^2y} dx \wedge dz + \sin(xyz) dx \wedge dy$$

Then

$$\begin{aligned} d\omega &= \frac{\partial e^{x^2y}}{\partial x} dx \wedge dx \wedge dz + \frac{\partial e^{x^2y}}{\partial y} dy \wedge dx \wedge dz + \frac{\partial e^{x^2y}}{\partial z} dz \wedge dx \wedge dz \\ &+ \frac{\partial \sin(xyz)}{\partial x} dx \wedge dx \wedge dy + \frac{\partial \sin(xyz)}{\partial y} dy \wedge dx \wedge dy + \frac{\partial \sin(xyz)}{\partial z} dz \wedge dx \wedge dy \\ &= 0 + x^2 e^{x^2y} dy \wedge dx \wedge dz + 0 + 0 + 0 + xy \cos(xyz) dz \wedge dx \wedge dy \\ &= (-x^2 e^{x^2y} + xy \cos(xyz)) dx \wedge dy \wedge dz \end{aligned}$$

**Contraction (inner product) and Lie derivative:**

$$X = e^x \frac{\partial}{\partial y} + (y^3 + \sin x) \frac{\partial}{\partial z}, \quad \alpha = (x^2 + y^2 + z^2) dx \wedge dz$$

Then

$$i_X \alpha = -(y^3 + \sin x)(x^2 + y^2 + z^2) dx, \quad d\alpha = 2y dy \wedge dx \wedge dz = -2y dx \wedge dy \wedge dz$$

(there is a typo in the sign of  $d\alpha$  in the lecture notes), and

$$\begin{aligned} L_X \alpha &= (i_X d + di_X) \alpha = i_X d\alpha + di_X \alpha \\ &= i_X (-2y dx \wedge dy \wedge dz) + d(-(y^3 + \sin x)(x^2 + y^2 + z^2) dx) \\ &= 2e^x y dx \wedge dz - 2y(y^3 + \sin x) dx \wedge dy \\ &+ (-3y^2(x^2 + y^2 + z^2) - 2y(y^3 + \sin x)) dy \wedge dx - 2z(y^3 + \sin x) dz \wedge dx \\ &= (2e^x y + 2z(y^3 + \sin x)) dx \wedge dz + 3y^2(x^2 + y^2 + z^2) dx \wedge dy. \end{aligned}$$

**Pull-backs:**

For  $F : \mathbb{R}^3 \rightarrow \mathbb{R}^2$  given by  $(x, y, z) \mapsto (u, v) = (y^2z, xy)$  the pull-back of the 2-form  $e^u du \wedge dv$  is

$$\begin{aligned} F^*(e^u du \wedge dv) &= e^{y^2z} d(y^2z) \wedge d(xy) \\ &= e^{y^2z} (2yz dy + y^2 dz) \wedge (x dy + y dx) \\ &= e^{y^2z} (-2y^2z dx \wedge dy - xy^2 dy \wedge dz - y^3 dx \wedge dz) \end{aligned}$$