Problem #1:

a) Find the integral \( \int_{\gamma} \alpha \) of
\[ \alpha = dz + ydx \in \Omega^1(\mathbb{R}^3) \]
along the path
\[ \gamma: [0, 2\pi] \to \mathbb{R}^3, \quad \gamma(t) = (\cos t, \sin t, t^2). \]

b) Is \( \alpha \) closed? exact? Explain briefly.

Problem #2:

a) Show that the function
\[ \Phi(t, x) = \left(3\sqrt{x} + t\right)^3, \]
is the flow \( \Phi_t(x) = \Phi(t, x) \) of a vector field on \( \{x | x \neq 0\} \subset \mathbb{R} \).

b) Find the vector field on \( \{x | x \neq 0\} \subset \mathbb{R} \) having the flow \( \Phi_t(x) \) from part a).

c) Is this vector field complete? Explain briefly.

Problem #3:

Consider the following coordinate transformation on \( \mathbb{R}^2 \),
\[ u = -2x - 7y, \quad v = x + 5y. \]
Express the coordinate vector fields
\[ \partial/\partial x, \partial/\partial y \]
for the \((x, y)\) coordinates in terms of the coordinate vector fields
\[ \partial/\partial u, \partial/\partial v \]
for the \((u, v)\) coordinates.

Problem #4: Consider the following vector fields on \( \mathbb{R}^2 \setminus \{0\} \),
\[ X = -y \frac{\partial}{\partial x} + x \frac{\partial}{\partial y}, \quad Y = x \frac{\partial}{\partial x} + y \frac{\partial}{\partial y}. \]
Find a 1-form \( \alpha \in \Omega^1(\mathbb{R}^2 \setminus \{0\}) \) such that
\[ \alpha(X) = 0, \quad \alpha(Y) = -1. \]
Problem #5:

a) Compute the Lie brackets $[X,Y]$, $[[X,Y],Y]$, and $[[[X,Y],Y],Y]$ of the following two vector fields on $\mathbb{R}^3$.

$$X = \frac{\partial}{\partial z} + y^3 \frac{\partial}{\partial x}, \quad Y = \frac{\partial}{\partial y}.$$ 

b) Explain briefly if there can be a 2-dimensional submanifold $S \subset \mathbb{R}^3$ such that $X, Y$ are everywhere tangent to $S$. 
