MAT 367S – Mock Midterm Test #2

4:00 – 4:50 (+20min to upload your work), March 24, 2020

`No tools allowed.  The test will not be graded.`

Problem #1:

a) Find the integral $\int_\gamma \alpha$ of

$$\alpha = \mathrm{d}z + y\mathrm{d}x \in \Omega^1(\mathbb{R}^3)$$

along the path

$$\gamma\colon [0, 2\pi] \to \mathbb{R}^3, \quad \gamma(t) = (\cos t,\, \sin t,\, t^2).$$

b) Is α closed? exact? Explain **briefly**.

Problem #2:

a) Show that the function

$$\Phi(t, x) = \left(\sqrt[3]{x} + t\right)^3,$$

is the flow $\Phi_t(x) = \Phi(t, x)$ of a vector field on $\{x|\ x \neq 0\} \subset \mathbb{R}$.

b) Find the vector field on $\{x|\ x \neq 0\} \subset \mathbb{R}$ having the flow $\Phi_t(x)$ from part a).

c) Is this vector field complete? Explain **briefly**.

Problem #3:

Consider the following coordinate transformation on $\mathbb{R}^2$,

$$u = -2x - 7y, \quad v = x + 5y.$$

Express the coordinate vector fields

$$\partial/\partial x,\ \partial/\partial y$$

for the (x, y) coordinates in terms of the coordinate vector fields

$$\partial/\partial u,\ \partial/\partial v$$

for the (u, v) coordinates.

Problem #4: Consider the following vector fields on $\mathbb{R}^2 \setminus \{0\}$,

$$X = -y\frac{\partial}{\partial x} + x\frac{\partial}{\partial y}, \quad Y = x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y}.$$

Find a 1-form $\alpha \in \Omega^1(\mathbb{R}^2 \setminus \{0\})$ such that

$$\alpha(X) = 0, \quad \alpha(Y) = -1.$$

Turn over ===>

Problem #5:

a) Compute the Lie brackets $[X, Y]$, $[[X, Y], Y]$, and $[[[X, Y], Y], Y]$ of the following two vector fields on $\mathbb{R}^3$.

$$X = \frac{\partial}{\partial z} + y^3 \frac{\partial}{\partial x}, \quad Y = \frac{\partial}{\partial y}.$$

b) Explain **briefly** if there can be a 2-dimensional submanifold $S \subset \mathbb{R}^3$ such that X, Y are everywhere tangent to S.