(the differentials dF and dG, taken at point M, lie in the closure of $\mathfrak{G}$ and their commutator is defined).

The Euler equation preserves the orbits of the coadjoint representation of $\mathfrak{G}$ and is a Hamiltonian equation with Hamiltonian $H(M) = \langle M, A^{-1}M\rangle$, which is called the energy.

2. Recall that the Virasoro algebra V is the unique nontrivial central extension by means of R of the Lie algebra Vect S^1 (of vector fields of the circle). Its elements can be identified with the pairs (2π-periodic function, number). Then a commutator in V takes the form

$$[(f(x), a), (g(x), b)] = (f(x)g'(x) - g(x)f'(x), \int f'(x)g''(x)\,dx)$$

(here and below the integration is over interval $[0, 2\pi]$).

Space V* can be identified with pairs (2π-periodic function, number). Bracket (1) on functions on V* is given by the formula

$$\{F, G\}(u(x), c) = \int \left[\left(\frac{\delta F}{\delta u}\left(\frac{\delta G}{\delta u}\right)' - \frac{\delta G}{\delta u}\left(\frac{\delta F}{\delta u}\right)'\right) u + c\left(\frac{\delta F}{\delta u}\right)'\left(\frac{\delta G}{\delta u}\right)''\right] dx \tag{1'}$$

(as functions on V* it is sufficient to consider integrals of differential polynomials (see [2])), $\delta F/\delta u(x)$ is defined by the equation

$$d/d\varepsilon F(u + \varepsilon v, c)|_{\varepsilon=0} = \int \frac{\delta F}{\delta u}(x)\, v(x)\, dx.$$

The Hamiltonian equation with Hamiltonian F takes the form

$$\dot{u} = 2\left(\frac{\delta F}{\delta u}\right)' u + \frac{\delta F}{\delta u} u' - c\left(\frac{\delta F}{\delta u}\right)''', \quad \dot{c} = 0. \tag{2'}$$

Consider inertia operator A such that $A(f, a) = (f, a) \in V^*$. It defines a scalar product on V: $((f, a), (g, b)) = \int fg\,dx + ab$.

<u>Proposition 1.</u> The Euler equation corresponding to inertia operator A coincides with the KdV equation.

<u>Proof</u>. The energy on V* equals $H(u, c) = {}^1/_2 \int u^2(x)dx + c^2/2$. The Hamiltonian equation

$$\dot{u} = 3u'u - cu''' \tag{3}$$

corresponds to it.

3. The Neveu–Schwarz (NS) and Ramond (R) superalgebras are the simplest superanalogues of the Virasoro algebra. They enter into a number of so-called Lie superalgebras of string theories [4]. The even parts of NS and R coincide with the Virasoro algebra, and the odd parts can be identified with functions of one variable such as $\psi(x + 2\pi) = -\psi(x)$ for NS and $\psi(x + 2\pi) = \psi(x)$ for R. A commutator in NS and R takes the form

$$[(f, \varphi, a), (g, \psi, b)] = (fg' - gf' + \varphi\psi/2, f\psi' - \psi f'/2 - g\varphi' + \varphi g'/2, \int (f'g'' + \varphi'\psi'/2)).$$

The dual spaces NS* and R* can be identified with a set of triples $(u(x), \xi(x), c)$, where $u(x)$ is a function on S^1 with values in the even part, and $\xi(x)$ is a function such that $\xi(x + 2\pi) = -\xi(x)$ (respectively, $\xi(x)$) with values in the odd part of some supercommutative ring.

Bracket (1) on NS* and R* (see [5; 6]) is defined by operator

$$P(u, \xi, c) = \begin{pmatrix} c\partial^3 - u\partial - \partial u & -\partial\xi/2 - \xi\partial \\ -\partial\xi - \xi\partial/2 & (c\partial^2 - u)/2 \end{pmatrix},$$

where $(u, \xi, c) \in NS^*(R^*)$, $\partial = d/dx$, and takes the form

$$\{F, G\}(u, \xi, c) = \left((\delta F/\delta u, \delta F/\delta\xi), P\begin{pmatrix}\delta G/\delta u \\ \delta G/\delta\xi\end{pmatrix}\right). \tag{1''}$$

The Hamiltonian equation with Hamiltonian F is defined by formula

$$\begin{pmatrix} \dot{u} \\ \dot{\xi} \end{pmatrix} = -P \begin{pmatrix} \delta F/\delta u \\ \delta F/\delta \xi \end{pmatrix}. \tag{2''}$$

Consider inertia operator A_s: $NS \to NS^*$ $(R \to R^*)$:

$$A_s(f(x), \varphi(x), a) = (f(x), {}^1/_4\partial^{-1}\varphi(x), c).$$

In the case of the NS superalgebra, it is uniquely defined and is nondegenerate, since integration operator ∂^{-1} acts in the space of functions with null average. For the R superalgebra, ∂^{-1} is defined by the formula $(\partial^{-1}u)(x) = \int\limits_0^x \left(u - \int u\right) dy - \int \int\limits_0^x \left(u - \int u\right) dy$. The corresponding metric proves to be degenerate.

Proposition 2. The Euler equation corresponding to inertia operator A_s coincides with the Korteweg–de Vries superequation from [6].

Proof. The energy equals $H(u, \xi, c) = {}^1/_2\int(u^2(x) = 4\xi'(x)\xi(x))dx + c^2/2$. By formula (2''), the Hamiltonian equation with Hamiltonian H has the form

$$\dot{u} = 3u'u - cu''' - 6\xi''\xi,$$
$$\dot{\xi} = 3u\xi' + 3u'\xi/2 - 2c\xi'''.$$

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