(the differentials dF and dG, taken at point M, lie in the closure of @ and their commutator
is defined).

The Euler equation preserves the orbits of the coadjoint representation of @ and is a
Hamiltonian equation with Hamiltonian H(M) = cRM, A^*HM, which is called the energy.

2. Recall that the Virasoro algebra V is the unique nontrivial central extension by
means of R of the Lie algebra Vect S^1 (of vector fields of the circle). Its elements can be
identified with the pairs (2π-periodic function, number). Then a commutator in V takes the
form

\[ [f(x), a] \cdot [g(x), b] = \int (f(x) \xi'(x) - g(x) \xi''(x)) \, dx \]

(here and below the integration is over interval [0, 2π]).

Space V^* can be identified with pairs (2π-periodic function, number). Bracket (1) on
functions on V^* is given by the formula

\[ \{F, G\} (u(x), c) = \int \left( \frac{\delta F}{\delta u} \frac{\delta G}{\delta u} - \frac{\delta G}{\delta u} \frac{\delta F}{\delta u} \right) u + c \left( \frac{\delta F}{\delta u} \right)' \frac{\delta G}{\delta u} \, dx \]  

(1')

(as functions on V^* it is sufficient to consider integrals of differential polynomials (see [2])).
\[ \delta F/\delta u(x) \] is defined by the equation

\[ \frac{d}{dx} \frac{\delta F}{\delta u}(u + ev, c) \bigg|_{u=0} = \int \frac{\delta F}{\delta u}(x) v(x) \, dx. \]

The Hamiltonian equation with Hamiltonian F takes the form

\[ u = 2 \frac{\delta F}{\delta u} u' + \frac{\delta F}{\delta u} u'' - c \frac{\delta F}{\delta u} \omega, \quad \omega = 0. \]  

(2')

Consider inertia operator A such that A(f, a) = (f, a) \equiv V^*. It defines a scalar product
on V: \((f, a), (g, b)\) = \( \int fgdx + ab \).

Proposition 1. The Euler equation corresponding to inertia operator A coincides with
the KdV equation.

Proof. The energy on V^* equals H(u, c) = \( \frac{1}{2} \int u^2(x) \, dx + \frac{c^2}{2} \). The Hamiltonian equation

\[ \dot{u} = 3u' - cu'' \]  

(3)

corresponds to it.

3. The Neveu–Schwarz (NS) and Ramond (R) superalgebras are the simplest superanallogues
of the Virasoro algebra. They enter into a number of so-called Lie superalgebras of string
theories [4]. The even parts of NS and R coincide with the Virasoro algebra, and the odd
parts can be identified with functions of one variable such as \( \psi(x + 2\pi) = -\psi(x) \) for NS and
\( \psi(x + 2\pi) = \psi(x) \) for R. A commutator in NS and R takes the form

\[ [f(x), \psi, (x), \psi', (x), (f'(x) + \psi'(x)/2)]. \]

The dual spaces NS^* and R^* can be identified with a set of triples \( (u(x), \xi(x), c) \),
where u(x) is a function on S^1 with values in the even part, and \( \xi(x) \) is a function such
that \( \xi(x + 2\pi) = -\xi(x) \) (respectively, \( \xi(x) \)) with values in the odd part of some super-
commutative ring.

Bracket (1) on NS^* and R^* (see [5; 6]) is defined by operator

\[ P(u, \xi, c) = \left( \begin{array}{cc} 2\xi' u' - x & u \xi' - 2 \xi' x \xi' \\ -u' \xi' & 2 \xi' \end{array} \right), \]

where \((u, \xi, c) \equiv NS^*(R^*)\), \( \delta = d/dx \), and takes the form

\[ \{F, G\} (u(x), c) = \left( \frac{\partial F}{\partial u}, \frac{\partial F}{\partial \xi} \right) \cdot \left( \frac{\partial G}{\partial u}, \frac{\partial G}{\partial \xi} \right) \].  

(1'')

The Hamiltonian equation with Hamiltonian F is defined by formula

\[ \dot{u} = 3u'-cu'' \]  

(3)
\[
\left( \frac{\dot{u}}{\xi} \right) = -p \left( \frac{\delta F/\delta u}{\delta F/\delta \xi} \right).
\]

Consider inertia operator \(A_g\): \(NS \rightarrow NS^\ast (R \rightarrow R^8)\):

\[A_g(f(x), \varphi(x, \omega)) = (f(x), 1/2 \delta^{-1} \varphi(x, \omega)).\]

In the case of the NS superalgebra, it is uniquely defined and is nondegenerate, since integration operator \(\delta^{-1}\) acts in the space of functions with null average. For the R superalgebra, \(\delta^{-1}\) is defined by the formula \(\delta^{-1}w(x) = \frac{x}{2} \int (u - \int u) dy - \int \int (u - \int u) dy\). The corresponding metric proves to be degenerate.

**Proposition 2.** The Euler equation corresponding to inertia operator \(A_g\) coincides with the Korteweg–de Vries superequation from [6].

**Proof.** The energy equals \(H(u, \xi, c) = \int f(u^2(x) = 4\xi'(x)(x))^2 dx + c^2/2\). By formula (2''), the Hamiltonian equation with Hamiltonian \(H\) has the form

\[
\dot{u} = 3u' - cu'' - 6\xi''\xi, \\
\dot{\xi} = 3u'\xi'' + 3u'\xi''/2 - 2c\xi''.'
\]

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