ERRATUM FOR "MOMENT MAPS, COBORDISMS, AND HAMILTONIAN GROUP ACTIONS"

Posted on October 3, 2014.

This is an erratum for my joint book with Viktor L. Ginzburg (U.C. Santa Cruz) and Victor Guillemin (M.I.T). The book appeared in 2002 in the series American Mathematical Society Mathematical Surveys and Monographs **98**.

Thanks to all those who called our attention to these errors.

A particularly huge thanks to Matthias Franz for detecting several subtle errors in connection with equivariant cohomology, for suggesting fixes, for providing updates, and for encouraging us to post an erratum.

I would be happy to receive suggestions for any corrections or additions (such as relevant citations) to this erratum; please send these to me at `karshon@math.toronto.edu`. Thanks,

Yael Karshon.

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p.54, proof of Proposition 4.14: our references to items (1)–(7) of Proposition 4.14 all appear as (4.14). (Thanks to Fabian Ziltener for noting this.)

Here are the corrected references:

- p.54, the line after Equation (4.18): "This proves (2)".
- 5 lines later: "Therefore, (1) implies (2) and (3)".
- 7 lines later" "Hence, (2) implies (1)".
- lines 5-6 from the bottom: "Condition (5) is proved. Condition (6) follows immediately".
- lines 2–4 from the bottom: "We have established the implications (1) $\Leftrightarrow$ (2) $\Rightarrow$ (3) $\Rightarrow$ (4) $\Rightarrow$ (5) $\Rightarrow$ (6)". Let us now show that the negation of (2) implies the negation of (6)".
- p.55 l.4–5: "Having shown the equivalence of Conditions (1)–(6), it remains to show that these conditions are equivalent to Condition (7)".
- 7-8 lines later: "Hence, the negation of (7) implies the negation of (2). Finally, assuming the negation of (2) . . .".
- 5–6 lines later: "This proves the negation of (7)".

Page 161: some lines are missing from the third figure.

Throughout Appendix C: we need to assume that the compact Lie group G is connected. In particular this assumption is needed in the following locations:

- Section 4.1 (p.206), when we say that the E_2 term of the spectral sequence is given by the tensor of the ordinary cohomology with $\mathcal{R}_G$;
- in Proposition C.26 (p.207) (e.g., take $M = G = S^1 \rtimes \mathbb{Z}_2$); and
- in Theorem C.35 (p.210) (e.g., take $M = G =$ a nontrivial finite group).

(Thanks to Matthias Franz.)

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p.207: Proposition C.25 and Corollary C.27 are false: In Proposition C.25, "torsion free" should be replaced by "free". In Corollary C.27, formality is only equivalent to the first condition; the 2nd and 3rd conditions are equivalent to each other, and follow from formality, but do not imply formality.

Freeness over $\mathcal{R}_G$ implies torsion-freeness over $\mathcal{R}_G$, but the converse is not true. Counterexamples were given by Matthias Franz and Volke Puppe in "Freeness of equivariant cohomology and mutants of compactified representations", pp. 87–98, in: M. Harada et al. (eds.), Toric Topology (Osaka 2006), Contemp. Math. 460, AMS, Providence, RI 2008.

Torsion-freeness does imply freeness for actions of S^1 or of SU(2), and for actions of $(S^1)^2$ on a compact orientable manifold (by Allday). Franz and Puppe's counterexamples, resolving a question posed by Allday, are $(S^1)^2$ acting on $\mathbb{CP}^1 \times \mathbb{CP}^1$ with two "opposite" fixed points removed, and higher rank tori acting on compact orientable manifolds.

This mistake was found by Oliver Goertsches and Dirk Töben from the University of Cologne and was communicated and explained to us by Matthias Franz.

Appendix C, Theorem C.20 (p.204): "Ad-invariant" is redundant, since G is abelian. (But "Ad-invariant" *is* needed in the generalization to non-abelian groups in Theorem C.70.) (Thanks to M.Franz.)

Appendix C, section 7.1, Theorem C.53: *compact manifold M* should be *compact oriented manifold M*.

Appendix C, section 9, 2nd paragraph: we say "Let $C_k(X)$ be the free group with generators $f\colon \Sigma \to X$". Instead, $C_k(X)$ should be the *quotient* of the free *abelian* group

with generators $f \colon \Sigma \to X$ by the subgroup that is generated by the elements of the form $(f + g - h)$, where f and g are maps to X and where h is the disjoint union of f and g. (Thanks to Haggai Teneh for questioning this definition.)

Appendix C, updates: (Thanks to M.Franz.)

First paragraph of section 8.2 (p.223): Poincaré duality was extended to the torus-equivariant setting by C. Allday, M. Franz, and V. Puppe, in "Equivariant cohomology, syzygies, and orbit structure", arXiv:1111.0957, to appear in Trans. Amer. Math. Soc. They defined an equivariant homology that is different from the homology of the Borel construction. (It's the homology of the $\mathcal{R}_T$-dual of the Cartan model.) Also see the theorem in [GS8, p.169] that is attributed to Metzler.

Last paragraph of section 8.2 (p.224): the question raised there has been answered in the negative in Remark 5.11 of the aforementioned paper by Allday-Franz-Puppe.

Section 8.3 Remark C.72 (p.226): the question raised there has been answered in the positive by O. Goertsches and S. Rollenske in "Torsion in equivariant cohomology and Cohen-Macaulay G-actions", Transformation Groups 16 (2011), 1063-1080: in Cor. 3.5 they show that $H_G^*(M^{\max})$ is free over $\mathcal{R}_G$.

Appendix C, proof of Theorem C.70 (p.225): We omit details of the injectivity modulo $\mathcal{R}_G$ torsion. For this we have to show that if an element u is $\mathcal{R}_T$–torsion then it is also $\mathcal{R}_G$–torsion. This can be done by passing from $f \in \mathcal{R}_T$ to $F \in \mathcal{R}_G$ similar to what is done in the proof of surjectivity. (Thanks to Matthias Franz.)

Appendix D on Spin^c structures: Reyer Sjamaar noted a problem in section 2.7. He says that we define the twist of a Spin^c structure P by a Hermitian line bundle L to be $P' = P \times_K U(L)$ where $U(L)$ is the unit circle bundle of L and K is the kernel of $Spin^c \to SO$. He writes this:

> Instead of taking the quotient of $P \times U(L)$ by K, shouldn't you take the quotient of $P \times U(L)$ by $M \times K$? This is not a quotient in the usual sense, but a quotient in the category of "manifolds over M". A group object in this category is a "Lie group over M". An example of a group object is the adjoint bundle,
>
> $$\mathrm{Ad}(P) = P \times_G G,$$
>
> where G acts on itself by conjugation. (The sections of $\mathrm{Ad}(P)$ are the gauge transformations of P.) The centre of $\mathrm{Ad}(P)$ is the "constant" group object $M \times K$. One can define actions of "groups over M" on "manifolds over M",

and quotients by such actions. For instance $M \times K$ acts on $P \times U(L)$ and the quotient is the "twist" of P by L.

This twisting simply amounts to replacing each fibre P_m of P by $P_m \times_K U(L_m)$.

Appendix G on non-degenerate abstract moment maps: a long time ago Megumi Harada noted a gap in our proof of Kirwan surjectivity for a torus. Also compare with a proof of Rebecca Goldin (which might too has a gap, according to a paper by Baird-Lin). Baird-Lin claim to prove Kirwan surjectivity for torus actions for an abstract moment map in the presence of an invariant almost complex structure.