

Guidelines for the first term test.

Study material:

- Spivak: Chapters 1–6, the assigned Appendix 1 of Chapter 4, and the parts that were assigned from Chapters 8 and 25.
- The handouts, including the parts whose proofs are left to the reader as exercises.
- Your notes from lectures, tutorials, office hours.
- My notes from lectures.
- The weightless assignments.
- The for-credit assignments.

If you will need to use the axioms for the real numbers, then I will provide them.

Topics:

- Axioms for the real numbers and their consequences
- Approximations for addition, multiplication, inversion, square root
- Natural numbers and induction
- The least upper bound axiom
- Functions and graphs
- Limits and continuity

Tentative large list of relevant terms.

- “if”, “only if”. “for all”, “exists”.
- You must be able to negate statements.
- Associativity, commutativity, distributivity for addition/multiplication; additive/multiplicative inverse.
- Properties of $<$ and \leq : trichotomy, anti-symmetry, transitivity; relation with addition/multiplication.
- Upper/lower bound for a set. A set being bounded from above / bounded from below / bounded. Maximum/minimum of a set. Least upper bound = supremum of a set; greatest lower bound = infimum of a set.
- Least upper bound property of the real numbers. Uniqueness of least upper bound. Similar properties for greatest lower bounds.
- Archimedean property of the real numbers (for every $x \in \mathbb{R}$ there is $n \in \mathbb{N}$ such that $n > x$).
- Absolute value. Distance in \mathbb{R} . Distance in \mathbb{R}^2 .
- Natural numbers = \mathbb{N} ; integers = \mathbb{Z} ; rational numbers = \mathbb{Q} .
- Recursive definition; inductive proof.
- Well ordering principle; principle of induction.
- ϵ -neighbourhood; punctured ϵ -neighbourhood; left/right ϵ -neighbourhood.
- Open/closed interval.
- Intersection/union/difference of sets. Complement of a set.
- Sum/product/composition of functions.
- Polynomial function. Degree/coefficients of a polynomial.

- Rational function.
- Dense subset of \mathbb{R} (a subset A such that every open interval contains an element of A)
- Trigonometric functions; trigonometric identities.
- Domain of a function. Graph of a function.
- Even/odd function. Periodic function.
- Characteristic function of a set.
- The floor function $\lfloor x \rfloor$.
- $\lim_{x \rightarrow a} f(x) = \ell$; $\lim_{x \rightarrow \infty} f(x) = \ell$; $\lim_{x \rightarrow a} f(x) = \infty$; $\lim_{x \rightarrow a} f(x) = \infty$; Left/right limit.
- Limit of a sum, of a product, of $1/f(x)$, of $\sqrt{f(x)}$.
- Continuity of a function at a point; left/right continuity at a point.
- Continuity of a sum/product/composition.
- Pathological examples of discontinuous functions.
- Continuous and positive implies positive in a neighbourhood.