APM 346: Problem set 2

Due Monday Oct 13,2003.

- 1. Solve Laplace's equation inside the rectangle $0 \leq x \leq L\,, 0 \leq y \leq H$, with the following boundary conditions

  (a) $\frac{\partial u}{\partial x}(0,y) = 0, \frac{\partial u}{\partial x}(L,y) = 0\,, u(x,0) = 0\,, u(x,H) = f(x)$

  (b)$\frac{\partial u}{\partial x}(0,y) = 0, \frac{\partial u}{\partial x}(L,y) = g(y)\,, u(x,0) = 0\,, u(x,H) = 0$

  (c)$\frac{\partial u}{\partial x}(0,y) = 0, \frac{\partial u}{\partial x}(L,y) = g(y)\,, u(x,0) = 0\,, u(x,H) = 0$

  (d)$u(0,y) = 0\,, u(L,y) = 0\,, u(x,0) - \frac{\partial u}{\partial y}(x,0) = 0\,, u(x,H) = f(x)$

- 2. Solve Laplace's equation outside a circular disk of radius R subject to the boundary conditions $u(R,\theta) = \ln 2 + 4\cos\theta$. Assume that $u(r,\theta)$ remains finite as $r \to \infty$. What is the solution for $u(r,\theta) = f(\theta)$ for an arbitrary function $f(\theta)$?

- 3. Solve Laplace's equation inside the quarter circle of radius $R = 1$ with $0 \leq \theta \leq \frac{1}{2}\pi$ for the following boundary values:

  (a) $\frac{\partial u}{\partial \theta}(r,0) = 0\,, u(r,\frac{1}{2}\pi) = 0\,, u(1,\theta) = f(\theta)$.

  (b) $\frac{\partial u}{\partial \theta}(r,0) = 0\,, \frac{\partial u}{\partial \theta}(r,\frac{1}{2}\pi) = 0\,, u(1,\theta) = f(\theta)$.

- 4. Solve Laplace's equation inside a circular annulus($a < r < b$) subject to the boundary conditions

  (a) $u(a,\theta) = f(\theta)\,, u(b,\theta) = g(\theta)$.

  (b) $u(a,\theta) = 0\,, u(b,\theta) = g(\theta)$.

  (c) $\frac{\partial u}{\partial r}(a,\theta) = f(\theta)\,, \frac{\partial u}{\partial r}(b,\theta) = g(\theta)$

- 5. Write the Fourier series in the interval $(-\pi,\pi)$ for the following functions:

  (a) $f(x) = x$ when $-\pi < x < \pi$. What is the sum of the series when $x = \pm\pi$?

  (b) $f(x) = -\pi$ when $\pi < x < 0$, and $f(x) = 0$ when $0 < x < \pi$. What is the sum of the series equal to when $x = 0$?

- 6. Write the Fourier series in the interval $(-\pi < x < \pi)$ for the following functions

  (a) $f(x) = \exp x$

  (b) $f(x) = \sinh x$