

## Math 246Y: Homework number 6

Due Monday, April 3, in class.

- (1) Prove that the closure of a set is always closed.
  - (2) Prove that the closure of the union of two sets is the union of their closures.
  - (3) Prove that a function is continuous if and only if the inverse image of every closed set is closed.
  - (4) Prove that a component of a topological space is a closed subset of the space.
  - (5) Prove that the number of components of a topological space is a topological invariant.
- (Extra Credit) What is the maximum number of distinct subsets of the real line that can be obtained by starting with a given subset and successively applying closure and complement (in any order)? Prove that your answer is correct.