MAT347Y1 HW9 Marking Scheme

Friday, November 27

Total: 21 points.

5.2.1: 5 points; 1 per part.

5.2.5: 3 points.

- (1) ($\Rightarrow$)
- (1) ($\Leftarrow$)
- (1) exponent n_1

A worryingly large proportion of you assumed (in some form or other) that $x^a = x^b$ implies $a = b$.

5.2.13: 4 points.

- (2) Presentation for A
- (1) there exists a homomorphism. In particular, your homomorphism must be well-defined! A (once again, worryingly) large proportion of people either forgot to check this entirely, or "checked" this in a meaningless way, like calculating that $\phi(a) = \phi(b)$ when $a = x_1^{i_1} \cdots x_n^{i_n} = b$. The whole point of proving something is well-defined is that if there are different ways of writing the same element (in this case, for instance, $x_1^2 = x_1^{n_1+2}$ and $x_1^3 x_2 = x_1 x_2 x_1^2$), those different ways of writing the element all give the same result in the image.

  Note that only this step that requires the condition on the g_i's having the right order; if you forget to prove your map is well-defined, you can do the whole question without using that information at all. That should set off warning bells in your head that you might be missing an important step.
- (1) uniqueness

5.2.14(a): 3 points: associative/commutative, identity, inverses

5.2.14(b): 6 points

- (2) χ_i each commute and have the right order
- (1) Define a homomorphism $G \to \hat{G}$ (e.g. using question 13)
- (2) The homomorphism is surjective
- (1) The homomorphism is injective

Note: a common mistake was to show that the χ_is commute, and conclude that $\langle \chi_1 \rangle \times \cdots \times \langle \chi_n \rangle \leq \hat{G}$ right after. This implication does not follow! The join of subgroups is not necessarily a direct product, even in the abelian case.