MAT347Y1 HW7 Marking Scheme

Friday, November 6

Total: 25 points.

4.3.24: 5 points.

- (1) Choose a maximal subgroup containing H. Make sure you understand what "maximal subgroup" means. In particular, there isn't only one, and not all maximal subgroups will contain H!
- (2) Case 1: Maximal subgroup not normal
- (2) Case 2: Maximal subgroup normal (Yes, this case is necessary. If you're going to use a result, make sure you check all the conditions!)

4.3.32: 4 points. Don't forget the class equation! $(1 + 2k + n)$.

Also, note that to show a set $S \subseteq G$ is a conjugacy class, it is not enough to show that given any $s \in S$, all conjugates of s are also in S. (In that case, G itself would be a conjugacy class!) You also need to show that given any two elements $s, t \in S$, that s and t are conjugates.

4.4.8:

(a) 2 points.

(b) 4 points (2 per proof).

(c) 2 points.

4.5.9: 6 points.

- (1) 3-Sylow subgroups have order 3
- (1) There are at most 4 Sylow 3-subgroups
- (4) Exhibit 4 Sylow 3-subgroups

Note: Many people tried to solve this problem by brute force: finding all solutions to $\left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right)^3 = \left(\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}\right)$ with $ad - bc = 1$ using some careful casework.

There's a cleaner way: use the Sylow Theorems! Once you deduce that Sylow 3-subgroups have order 3 (by computing the order of $\mathrm{SL}_2(\mathbb{F}_3)$), you only need to find a single element of order 3. Then Sylow Theorem part (2) tells you that all the other subgroups will be generated by conjugates of this element, and part (3) will tell you once you've found them all. Still some matrix computations needed, but this takes a lot less work than the brute force approach.

(Of course, the method of discovery doesn't even need to be included in the written solution; all that's needed is a complete list, and proof that it's a complete list)

5.1.5: 2 points.