MAT 347
An example of the FTGT
March 18, 2016

Let $f(X) = X^4 - 2$. Let K be the splitting field of $f(X)$ over $\mathbb{Q}$. Let $G = \mathrm{Gal}(K/\mathbb{Q})$.

1. Find all roots of $f(X)$ in $\mathbb{C}$.
2. Find a set of two elements that generate the field extension $K/\mathbb{Q}$.
3. Calculate $|K : \mathbb{Q}|$.
4. Find a basis for K as a $\mathbb{Q}$-vector space.
5. List all the elements of G by showing how they act on a set of generators.
6. Find all intermediate field extensions of $K/\mathbb{Q}$.
7. Which intermediate field extensions are normal? For each one of them, they have to be the splitting field of some polynomial. Find them.
8. Find a primitive element for the field extension $K/\mathbb{Q}$.