Total: 30 points.

1.7.19: 4 points.
• (1) Injectivity
• (1) Surjectivity
• (2) Lagrange’s Theorem

2.1.4: 3 points.

2.1.9: 4 points.
• (1) Nonempty subset (suffices to show it contains the identity)
• (3) Closed under $xy^{-1}$ (or under multiplication and inverses separately)

2.3.26:
(a) 3 points.
• (1) $\sigma_a$ is a homomorphism
• (1) $(a, n) = 1$ implies bijective
• (1) Bijective implies $(a, n) = 1$

(b) 2 points. (one for each direction)

(c) 2 points.

(d) 3 points.
• (1) $\sigma_a$ is well-defined
• (1) $\sigma_a \circ \sigma_b = \sigma_{ab}$ (so $\sigma_a \mapsto \sigma_a$ is a homomorphism)
• (1) $\sigma_a \mapsto \sigma_a$ is bijective (you need to show injectivity and surjectivity. Using “order considerations” here is circular reasoning, because you have to already know the map is a bijection in order to prove that the sets are the same size).

Handout #2: 9 points.
• (1) Classifying group elements based on the type of action (these will correspond to conjugacy classes in $S_4$).
• (4 for each problem) For each class, find how many diagrams are fixed, and apply Burnside’s Lemma.