MAT347Y1 HW2 Marking Scheme

Friday, October 2

Total: 16 points.

1.5.1: 3 points.

1.6.17: 3 points.

1.7.17: 5 points.

- (1) conjugation by g is a homomorphism
- (1) conjugation by g is injective
- (1) conjugation by g is surjective (Even though the domain and codomain have the same order, proving injectivity alone is not enough! Can you think of a group G and injective homomorphism $\phi : G \to G$ such that ϕ is not surjective?)
- (1) conjugation by g preserves order of subsets (because it is a bijection)
- (1) conjugation by g preserves order of elements (because it is an isomorphism - homomorphisms don't necessarily preserve order)

Note: it is possible to prove that conjugation by g (call the map ϕ) is an isomorphism by giving an inverse. This does *not* mean finding ψ with $\phi(x)\psi(x) = \psi(x)\phi(x) = 1$, this means finding ψ with $\psi(\phi(x)) = \phi(\psi(x)) = x$. Be careful what you're inverting.

1.7.21: 5 points. Let G be the group of rigid motions of the cube.

- (2) Define a homomorphism $\phi : G \to S_4$ (equivalently, a group action)
- One of the following:
  - (2) ϕ is injective (this was by far the biggest source of errors. Just because two rigid motions act the same way on pairs, how do you know they act the same way on the vertices *within* the pairs? Could a rigid motion swap vertices within some of the pairs while keeping the pairs themselves in the same place?)
  - (2) ϕ is surjective (e.g. by providing a generating set of S_4 in terms of images of ϕ)
- (1) Use the fact that $|G| = 24$ and injectivity/surjectivity to conclude that ϕ is a bijection