MAT347Y1 HW16 Marking Scheme
Friday, March 4

Total: 38 points.

#1: 18 points; 3 per part. One small caution: being able to construct an angle \( \theta \) does NOT mean that \( \theta \) is a constructible number. For instance, the angle \( \pi/4 \) is easy to construct, but \( \pi/4 \) is definitely not a constructible number.

#2: 5 points. A few things to note:
- \( |Gal(\mathbb{Q}(\zeta_p)/\mathbb{Q})| = p - 1 \) (not \( p \)).
- Don’t forget your group theory - what do you need to do to show that two groups are isomorphic?

#3: 15 points; 3 per part.
Part (d) caused a lot of people trouble. Here’s an outline of one way to do it:
- Let \( \phi \) be in the Galois group, and suppose \( \phi(X) = \frac{P(X)}{Q(X)} \) for some rational function in \( \mathbb{C}(X) \). Then for all \( r(X) \in \mathbb{C}(X) \), we have \( \phi(r(X)) = r \left( \frac{P(X)}{Q(X)} \right) \in \mathbb{C} \left( \frac{P(X)}{Q(X)} \right) \) (Why? Write out \( r(X) \) and use the fact that \( \phi \) is a field homomorphism fixing \( \mathbb{C} \))
- We want \( \phi \) to be surjective, so when is \( \mathbb{C} \left( \frac{P(X)}{Q(X)} \right) = \mathbb{C}(X) \)? (Use the hint)
- The conditions on \( P(X) \) and \( Q(X) \) from the hint can be turned into another equivalent condition, that a certain determinant is nonzero.