

MAT347Y1 HW16 Marking Scheme

Friday, March 4

Total: 38 points.

#1: 18 points; 3 per part. One small caution: being able to construct an angle θ does NOT mean that θ is a constructible number. For instance, the angle $\pi/4$ is easy to construct, but $\pi/4$ is definitely not a constructible number.

#2: 5 points. A few things to note:

- $|Gal(\mathbb{Q}(\zeta_p)/\mathbb{Q})| = p - 1$ (not p).
- Don't forget your group theory - what do you need to do to show that two groups are isomorphic?

#3: 15 points; 3 per part.

Part (d) caused a lot of people trouble. Here's an outline of one way to do it:

- Let ϕ be in the Galois group, and suppose $\phi(X) = \frac{P(X)}{Q(X)}$ for some rational function in $\mathbb{C}(X)$. Then for all $r(X) \in \mathbb{C}(X)$, we have $\phi(r(X)) = r\left(\frac{P(X)}{Q(X)}\right) \in \mathbb{C}\left(\frac{P(X)}{Q(X)}\right)$ (Why? Write out $r(X)$ and use the fact that ϕ is a field homomorphism fixing \mathbb{C})
- We want ϕ to be surjective, so when is $\mathbb{C}\left(\frac{P(X)}{Q(X)}\right) = \mathbb{C}(X)$? (Use the hint)
- The conditions on $P(X)$ and $Q(X)$ from the hint can be turned into another equivalent condition, that a certain determinant is nonzero.