MAT347Y1 HW14 Marking Scheme

Friday, February 12

Total: 33 points.

General note: an unfortunate (or fun, depending on your perspective) aspect of this part of the course is that there are many different but similar-sounding results, and mixing them up can get you into trouble. The two main instances from this HW:

- (9.2.2) Does the Euclidean algorithm give you a unique remainder? In general, NO (e.g. in $\mathbb{Z}$, dividing 43 by 5 is either 8 remainder $r = 3$ or 9 remainder $r = -2$; both values of r satisfy $|r| < |5|$). In polynomial rings, YES (Section 9.2, Theorem 3).
- (13.1.1) Is Eisenstein's Criterion sufficient to tell you $x^3 + 9x + 6 \in \mathbb{Q}[x]$ is irreducible? Using the standard formulation (Section 9.4, Proposition 13), NO (in $\mathbb{Q}$, (3) is the whole ring). Using another statement labeled as Eisenstein's Criterion (Corollary 14), YES.

In both cases, if you just applied the result as is, my policy was to take off points unless you were clear about which version you were using (because I'm not a mind-reader and so for all I know you were actually using the more general, and hence incorrect, version).

9.2.2: 4 points.

9.3.4:

(a) 4 points.

(b) 5 points: (1) degree 0 irreducibles are the primes of $/ZZ$, and (1) these are prime in R. (2) degree ≥ 1 irreducibles are irreducibles of $\mathbb{Q}[x]$ with constant term ± 1, and (1) these are prime in R.

(c) 3 points.

(d) 3 points. Describing a ring means more than just describing it as a set: what do addition and (more interestingly) multiplication do?

9.4.4: 6 points. Assume $f(x) = a(x)b(x)$:

- (1) For each $k = 1, \ldots, n$, either $a(k) = b(k) = 1$ or $a(k) = b(k) = -1$
- (2) Show $a(x) = b(x)$ (so $f(x)$ is a square)
- (1) $n = 2$ case
- (2) $n \geq 6$ case (either by finding x such that $f(x) < 0$, or by finding $1 \leq i, j \leq n$ with $|i - j| > 2$ and $a(i) - a(j) = \pm 2$)

9.4.7: 4 points. The simplest way is by the first isomorphism theorem ($\mathbb{R}[x] \to \mathbb{C}$ by $x \mapsto i$, what is the kernel?). If you try to define a bijection directly from $\mathbb{R}[x]/(x^2+1)$, you have to show it is well-defined (e.g. by by choosing a unique representative from each equivalence class and defining it on that).

13.1.1: 4 points.

- (2) Irreducibility (Gauss' Lemma, Eisenstein; or rational root).
- (2) Finding the root