

MAT347Y1 HW11 Marking Scheme

Friday, January 15

Total: 29 points.

7.3.28: 4 points.

- (2) If the characteristic is composite, we can write $0 = ab$
- (1) a and b are nonzero (so are therefore zero divisors)
- (1) characteristic can't be 1 (only six people checked this!)

7.3.29: 5 points.

- (1) Nonempty
- (1) Closed under multiplication by anything in R
- (3) Closed under subtraction (or addition and additive inverses)

7.4.7: 6 points.

- (3) $R[x] \rightarrow R : p \mapsto p(0)$ is a surjective ring homomorphism with kernel (x)
- (3) Apply First Isomorphism Theorem and Propositions 12 and 13

7.4.9: 5 points.

- Ideal: (1) nonempty, (1) multiplication by R , (1) subtraction or addition+inverse
- (2) Not prime ideal (exhibit counterexample)

7.4.31:

- (a) 3 points. Note: Many people solved this by saying essentially “factor over and over, and the algorithm will eventually terminate.” No marks removed for this, but it’s a lot cleaner to let n be the *smallest* natural number with $r^n = 0$, so that factoring it immediately gives a contradiction.
- (b) 6 points. 2 per direction of implication, 2 for the statement to deduce.