Total: 31 points.

7.1.14:
(a) 2 points. If you claim \( x \neq 0 \) is a zero divisor by saying \( xy = 0 \) for some \( y \), you also need \( y \neq 0 \).
(b) 1 point.
(c) 2 points.
(d) 2 points.

7.1.25:
(a) 2 points.
(b) 2 points.
(c) 3 points. Note that this ring is not commutative, so \( xy = 1 \) does not imply \( yx = 1 \) (there are rings in which some elements have an inverse on the right but none on the left - these are not units!)

7.3.12:
(a) 3 points (nonempty, subtraction, multiplication)
(b) 4 points (respects addition, respects multiplication, bijective)
(c) 5 points.

7.3.21: 5 points.
- (3) the set of matrix entries, \( J \), is an ideal.
- (2) \( M_n(J) = I \)

Note: If you want to show some \( J \subseteq R \) is an ideal, proving it’s a subring first is actually doing more work than necessary. You need to show \( ab \in J \) for all \( a, b \in J \) to show it’s a subring, but you eventually need to show \( ar, ra \in I \) for any \( a \in I, r \in R \), which implies the first statement. So instead of checking both, replace the “closed under multiplication” check with the “closed under multiplication by anything in \( R \)” check.