MAT 247, Winter 2014
Assignment 5
Due Feb 11

1. For each of the following pairs of polynomials $p(x), q(x)$, either find a matrix A whose minimal polynomial is $p(x)$ and whose characteristic polynomial is $q(x)$, or explain why no such matrix exists.

   (a) $p(x) = (x-1)(x-2), q(x) = (x-1)(x-2)^3$

   (b) $p(x) = (x-1)(x-2), q(x) = (x-1)(x-2)^2(x-3)$

   (c) $p(x) = (x-1)^3, q(x) = (x-1)^4$

   (d) $p(x) = (x-1)^2(x-2), q(x) = (x-1)(x-2)^2$

2. Consider the following matrix

$$A = \begin{bmatrix} 5 & 1 & -4 \\ -9 & -1 & 9 \\ 0 & 0 & 1 \end{bmatrix}$$

   (a) Find a Jordan form matrix similar to A.

   (b) Let $T : \mathbb{C}^3 \to \mathbb{C}^3$ denote the linear operator defined by $T(v) = Av$. Find a basis β for $\mathbb{C}^3$ such that $[T]_\beta$ is a Jordan form matrix.

3. Let $T : V \to V$ be a linear operator and let $v \in V$ be a non-zero vector. Let W be the span of $\{v, T(v), T^2(v), \dots\}$.

   (a) Show that W is a T-invariant subspace and that W is T-cyclic.

   (b) Let $p(x)$ denote the characteristic polynomial of $T|_W : W \to W$. Recall that in class we described $p(x)$ and proved that $p(T|_W) = 0$. Prove that $p(x)$ divides the characteristic polynomial of $T : V \to V$.

(c) Use this to give a different proof of the Cayley-Hamilton theorem.

4. Let $p(x) \in \mathbb{F}[x]$ be a monic polynomial.

   (a) Find a matrix A (with entries in $\mathbb{F}$) such that $\det(xI - A) = p(x)$.

   (b) Find a matrix A (with entries in $\mathbb{F}$) such that the minimal polynomial of A is $p(x)$.