1. Let $V, \langle \cdot, \cdot \rangle$ be an inner product space. Let $W \subset V$ be a subspace.
   
   (a) Give the definition of $W^\perp$, the orthogonal complement of $W$.
   
   (b) Suppose that $W^\perp = V$. Prove that $W = \{0\}$. 
2. Consider $\mathbb{R}^3$ with the usual inner product. Let $W$ be the span of $(1, 0, 0)$ and $(1, 1, 1)$.

   (a) Perform the Gram-Schmidt process to these vectors to find an orthonormal basis for $W$.
   
   (b) Find the orthogonal projection of $(0, 0, 1)$ onto $W$. 
3. Let $V$ be a real inner product space.

(a) Given the definition of a self-adjoint linear operator on $V$.

(b) Suppose that a linear operator $T : V \to V$ is orthogonally diagonalizable (i.e. there exists an orthonormal basis for $V$ consisting of eigenvectors for $T$). Show that $T$ is self-adjoint.
4. Let $V$ be an inner product space.

(a) Give an example of a linear operator $T : V \to V$ such that $\text{null}(T) \neq \text{null}(T^*)$.

(b) Show that it is not possible to find an example when $T$ is normal.

(c) Show that for any linear operator $T : V \to V$, $\dim \text{null}(T) = \dim \text{null}(T^*)$. 

5. Let \( V, \langle \cdot, \cdot \rangle \) be an inner product space and let \( T : V \to V \) be a linear operator. Suppose that for all pairs of vectors \( v, w \in V \), \( \langle Tv, Tw \rangle = 0 \) if and only if \( \langle v, w \rangle = 0 \) (in other words, \( T \) preserves the property of orthogonality). Show that there exists some scalar \( a \) such that \( aT \) is an isometry.