MAT 247
ASSIGNMENT 3
DUE THURSDAY FEBRUARY 3

(1) (Axler 6.29) Let V be an inner product space and let $T : V \to V$ be a linear map. Let U be a subspace of V. Show that U is invariant under T if and only if $U^\perp$ is invariant under T^*.

(2) (Axler 7.2) Prove or give a counterexample: the product of any two self-adjoint operators on a finite-dimensional inner product space is self-adjoint.

(3) (Axler 7.8) Show that there is no self-adjoint operator $T : \mathbb{R}^3 \to \mathbb{R}^3$ such that $T(1, 2, 3) = 0$ and $T(2, 5, 7) = (2, 5, 7)$.

(4) Consider the linear operator $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by the matrix $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$. Show that T is self-adjoint. Find an orthonormal basis for $\mathbb{R}^2$ consisting of eigenvectors for T.