ASSIGNMENT 5
DUE THURSDAY NOV 26

(1) Exercise IV.6 on page 85 of Perrin.

You will need to use Theorem 3.7 on page 78, which we did not cover in class. So the first thing to do is to read and understand this theorem.

Bonus: in class, we showed that the Grassmannian was a projective variety. We gave an explicit embedding into projective space. Use our embedding to show that the φ defined in this question is actually a morphism of varieties.

(2) Find the singular points on the variety in $\mathbb{P}^3$ defined by $xy^2 - z^2t$.

(3) Consider the variety X of nilpotent 2x2 matrices (ie the set of 2x2 matrices A such that $A^2 = 0$). Show that $A \in X$ is a singular point of X if and only if $A = 0$.

You may use (without proof) the fact that the ideal of X in k^4 (all matrices) is generated by the trace and determinant.

Bonus: Generalize this to 3x3 matrices. Show that $A \in X$ is a singular point if and only if $rank A \leq 1$ (here X is the variety of nilpotent 3x3 matrices). You may use that the ideal of X in k^9 is generated by the coefficients of the characteristic polynomial. You may also use that $dim(X) = 6$.

(4) Let $X \subset \mathbb{P}^n$ be a smooth projective variety which is not contained in any projective hyperplane and also is not equal to $\mathbb{P}^n$.

Let

$$C(X) = \{(a_0, \ldots, a_n) : [a_0, \ldots, a_n] \in X\} \cup \{0\} \subset k^{n+1}$$

be the cone on X (recall that it is an affine variety). Show that 0 is the only singular point of X.