ASSIGNMENT 1
DUE THURSDAY SEPTEMBER 24

(1) If I, J are ideals, show that $V(IJ) = V(I) \cup V(J)$.
(Recall that in class we already proved that $V(I \cap J) = V(I) \cup V(J)$ — why doesn't this contradict the Nullstellansatz.)

(2) Exercise I.4 from Perrin (page 24).

(3) Find the irreducible components of the algebraic variety in k^3 defined by the equations $y^2 = xz$ and $z^2 = y^3$.

(4) Let X be an irreducible topological space. Prove that any non-empty open subset U of X is dense. (This means that the smallest closed subset of X which contains U is all of X.)

Bonus Define X to be the set of pairs of matrices whose product in either direction is 0, ie.

$$X := \{(A, B) \in Mat_{n,m}(k) \times Mat_{m,n}(k) : AB = BA = 0\}$$

Show that X is an affine variety. Find the components of X.