ASSIGNMENT 3
DUE THURSDAY FEBRUARY 2

(1) Let $F$ be a field and consider the extension $F \subset F(t)$. The goal of this exercise is to study $Gal(F(t)/F)$.

(a) Suppose that $a, b, c, d \in F$ with $ad - bc \neq 0$. Prove that the map

$$f(t) \mapsto f\left(\frac{at + b}{ct + d}\right)$$

is an automorphism of $F(t)$.

(b) Prove that this gives a homomorphism of groups $GL_2(F) \to Gal(F(t)/F)$.

(c) Prove that this homomorphism is surjective.

(d) Show that the kernel of this homomorphism consists of multiples of the diagonal matrix.

(e) Use all this to find the size of $Gal(F(t)/F)$ when $F = \mathbb{F}_q$.

(2) Let $f(x)$ be an irreducible cubic polynomial over a field $F$. Suppose that $K$ is the splitting field of $f(x)$. What are the possibilities for $Gal(K/F)$?