Welcome to MAT137 - Calculus with proofs!

- Class begins at 11:10am ET
- Test 1 will be on October 23.
- Check the website for office hours.

- **Before next class:**
  - **Watch videos 2.12, 2.13**
  - Download next class slides.
    No need to look at them.

Product formula?

Let $a \in \mathbb{R}$. Let f and g be positive functions defined near a, except maybe at a.

Assume $\lim_{x \to a} f(x) = 0$.

Can we conclude that $\lim_{x \to a} f(x)g(x) = 0$?

Hint: Think of some examples!

A harder proof

Prove

$$\lim_{x \to 0} \left(x^3 + x^2\right) = 0$$

Let $\varepsilon > 0$. How should we choose δ?.

Rough work:

$$|x^3 + x^2| = |x|^2 |x + 1| < \varepsilon$$

What if there was no pesky $|x + 1|$?
If x is close to 0, how big can $|x + 1|$ be?

A theorem about limits

Let f be a function with domain $\mathbb{R}$ such that

$$\lim_{x \to 0} f(x) = 3$$

Prove that

$$\lim_{x \to 0} f(2x) = 3$$

directly from the definition of limit. Do not use any of the limit laws.

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1. Write down the formal definition of the statement you want to prove.
2. Write down what the structure of the formal proof should be, without filling the details.
3. Rough work.
4. Write down a complete proof.