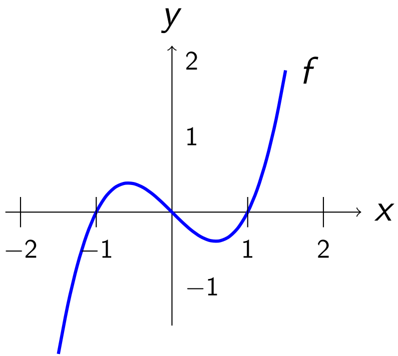
Welcome to MAT137 - Calculus with proofs!

- Assignment 3 is due on November 5.

- **Before next class:**
  - **Watch videos 4.3, 4.4**
  - Download next class slides.
    No need to look at them.

Finding a Restricted Domain on which a Function is Invertible



1. Find the largest interval containing 0 on which f is invertible.
2. Find the largest interval containing 1 on which f is invertible.

Absolute value and inverses

Let

$$h(x) = x|x| + 1$$

1. Calculate $h^{-1}(-8)$.

Absolute value and inverses

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$$h(x) = x|x| + 1$$

1. Calculate $h^{-1}(-8)$.
2. Sketch the graph of h.
3. What is the domain of h and what is its range?
4. On what interval is h invertible?
5. Find an equation for h^{-1}.
6. Sketch the graph of h^{-1}.

Composition and inverses

Let

$$f(x) = x + 1, \qquad g(x) = x^3.$$

We are interested in the composition $f \circ g$ and its inverse $(f \circ g)^{-1}$.

Which of the following statements is true?

1. $(f \circ g)(x)$ is "add one to x, then cube the result".
2. $(f \circ g)(x)$ is "cube x, then add one to the result".
3. $(f \circ g)^{-1}(x)$ is "substract one to x, then take the cube root of the result".
4. $(f \circ g)^{-1}(x)$ is "take the cube root of x, then subtract one from the result".

Composition and inverses

Assume for simplicity that all functions in this problem have domain $\mathbb{R}$.

Let f and g be functions. Assume they each have an inverse.

Is $(f \circ g)^{-1} = f^{-1} \circ g^{-1}$?

- If YES, prove it.
- If NO, fix the statement, then prove it!

Composition of one-to-one functions

Assume for simplicity that all functions in this problem have domain $\mathbb{R}$. Prove the following theorem.

Theorem

Let f and g be functions.
IF $f \circ g$ is one-to-one, THEN g is one-to-one.

Increasing and one-to-one

A function f is called *increasing* on an interval I, if for all $x_1, x_2 \in I$,

$$x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$$

Which of the following statements is true?

Theorem?

If f is increasing on I, then it is one-to-one on I.

Theorem?

If f is one-to-one on I, then it is increasing on I.