Welcome back to MAT137 - Calculus with proofs!

- Assignment 7 is due on February 25
- Assignment 8 is due on March 4
- Test 4 (Units 9–12) will open on March 12

- **Before next class:**
  - Watch videos 12.1, 12.4, 12.5.

Proof of Theorem 3

Theorem 3

Let $\{a_n\}_{n=0}^{\infty}$ be a sequence.

- IF $\{a_n\}_{n=0}^{\infty}$ is increasing AND unbounded above,
- THEN $\{a_n\}_{n=0}^{\infty}$ is divergent to ∞

1. Write the definitions of "increasing", "unbounded above", and "divergent to ∞"
2. Using the definition of what you want to prove, write down the structure of the formal proof.
3. Do some rough work if necessary.
4. Write a formal proof.

Proof feedback

1. Does your proof have the correct structure?
2. Are all your variables fixed (not quantified)? In the right order? Do you know what depends on what?
3. Is the proof self-contained? Or do I need to read the rough work to understand it?
4. Does each statement follow logically from previous statements?
5. Did you explain what you were doing? Would your reader be able to follow your thought process without reading your mind?

Critique this proof

- $\forall M \in \mathbb{R},\ \exists n_0 \in \mathbb{N},\ \forall n \in \mathbb{N}, \quad n \geq n_0 \implies x_n > M$

- M is not an upper bound: $\exists n_0 \in \mathbb{N}$ s.t. $a_{n_0} > M$

- $n \geq n_0 \implies a_n \geq a_{n_0} > M$

Calculations

1. $$\lim_{n\to\infty} \frac{n! + 2e^n}{3n! + 4e^n}$$

2. $$\lim_{n\to\infty} \frac{2^n + (2n)^2}{2^{n+1} + n^2}$$

3. $$\lim_{n\to\infty} \frac{5n^5 + 5^n + 5n!}{n^n}$$

True or False – The Big Theorem

Let $\{a_n\}_{n=0}^{\infty}$ and $\{b_n\}_{n=0}^{\infty}$ be positive sequences.

1. IF $a_n << b_n$, THEN $\forall m \in \mathbb{N},\ a_m < b_m$.

2. IF $a_n << b_n$, THEN $\exists m \in \mathbb{N}$ s.t. $a_m < b_m$.

3. IF $a_n << b_n$, THEN $\exists n_0 \in \mathbb{N}$ s.t.
   $\forall m \in \mathbb{N},\ m \geq n_0 \implies a_m < b_m$.

4. IF $\forall m \in \mathbb{N},\ a_m < b_m$, THEN $a_n << b_n$.

5. IF $\exists m \in \mathbb{N}$ s.t. $a_m < b_m$, THEN $a_n << b_n$.

6. IF $\exists n_0 \in \mathbb{N}$ s.t. $\forall m \in \mathbb{N},\ m \geq n_0 \implies a_m < b_m$,
   THEN $a_n << b_n$.