

Welcome back to MAT137 - Calculus with proofs!

- Assignment 7 is due on February 25
 - Assignment 8 is due on March 4
 - Test 4 (Units 9-12) will open on March 12
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- **Before next class:**
 - Watch video 11.7, 11.8.

A sequence defined by recurrence

Consider the sequence $\{R_n\}_{n=0}^{\infty}$ defined by

$$\begin{cases} R_0 = 1 \\ \forall n \in \mathbb{N}, & R_{n+1} = \frac{R_n + 2}{R_n + 3} \end{cases}$$

Compute R_1, R_2, R_3 .

Is this proof correct?

Claim:

$$\lim_{n \rightarrow \infty} R_n = -1 + \sqrt{3}.$$

Proof.

- Let $L = \lim_{n \rightarrow \infty} R_n$.
- $R_{n+1} = \frac{R_n + 2}{R_n + 3}$
- $\lim_{n \rightarrow \infty} R_{n+1} = \lim_{n \rightarrow \infty} \frac{R_n + 2}{R_n + 3}$
- $L = \frac{L + 2}{L + 3}$
- $L(L + 3) = L + 2$
- $L^2 + 2L - 2 = 0$
- $L = -1 \pm \sqrt{3}$
- L must be positive, so
 $L = -1 + \sqrt{3}$



Another sequence defined by recurrence

Consider the sequence $\{a_n\}_{n=0}^{\infty}$ defined by

$$\begin{cases} a_0 = 1 \\ \forall n \in \mathbb{N}, & a_{n+1} = 1 - a_n \end{cases}$$

- Use the same method as in the previous slide to compute its limit.
- **After** you have computed the limit, calculate a_2 , a_3 , a_4 , and a_5 .
- What happened?

The original sequence defined by recurrence – done right

Consider the sequence $\{R_n\}_{n=0}^{\infty}$ defined by

$$\begin{cases} R_0 = 1 \\ \forall n \in \mathbb{N}, & R_{n+1} = \frac{R_n + 2}{R_n + 3} \end{cases}$$

1. Prove $\{R_n\}_{n=0}^{\infty}$ is bounded below by 0.
2. Prove $\{R_n\}_{n=0}^{\infty}$ is decreasing.
3. Prove $\{R_n\}_{n=0}^{\infty}$ is convergent (use a theorem).
4. Now the calculation in the earlier slide is correct, and we can get the value of the limit.

Theorem 3

Let $\{a_n\}_{n=0}^{\infty}$ be a sequence.

- IF $\{a_n\}_{n=0}^{\infty}$ is increasing AND unbounded above,
- THEN $\{a_n\}_{n=0}^{\infty}$ is divergent to ∞

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1. Write the definitions of “increasing”, “unbounded above”, and “divergent to ∞ ”
2. Using the definition of what you want to prove, write down the structure of the formal proof.
3. Do some rough work if necessary.
4. Write a formal proof.

1. Does your proof have the correct structure?
2. Are all your variables fixed (not quantified)? In the right order? Do you know what depends on what?
3. Is the proof self-contained? Or do I need to read the rough work to understand it?
4. Does each statement follow logically from previous statements?
5. Did you explain what you were doing? Would your reader be able to follow your thought process without reading your mind?

Critique this proof

- $\forall M \in \mathbb{R}, \exists n_0 \in \mathbb{N}, \forall n \in \mathbb{N}, n \geq n_0 \implies x_n > M$
- M is not an upper bound: $\exists n_0 \in \mathbb{N}$ s.t. $x_{n_0} > M$
- $n \geq n_0 \implies x_n \geq x_{n_0} > M$