

Welcome back to MAT137 - Calculus with proofs!

- Assignment 10 is due on April 8. You have all the knowledge to start this assignment now.
- Test 5 will be on April 22 (Thursday).

- **Before next class:**
 - Watch videos 14.12, 14.14.

Taylor series gymnastics

Write the following functions as power series centered at 0. Write them first with sigma notation, and then write out the first few terms.

1. $f(x) = e^{-x}$

2. $f(x) = x^2 \cos x$

3. $f(x) = \frac{1}{1-x^2}$

4. $f(x) = \frac{x}{3+2x}$

5. $f(x) = \sin(2x^3)$

6. $f(x) = \frac{e^x + e^{-x}}{2}$

Note: You do not need to take any derivatives. You can reduce them all to other Maclaurin series you know.

Once you find the Maclaurin series, find $f^{(22)}(0)$.

Write the Taylor series...

1. for $f(x) = e^x$ at $a = -1$
2. for $g(x) = \sin x$ at $a = \pi/4$

You can do these problems in two ways:

1. Compute first few derivatives, guess the pattern, use general formula
2. Use substitution $u = x - a$, use known Maclaurin series (without computing any derivative).

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Hint: Recall that $G'(x) = \frac{1}{1+x^2}$. Then use the geometric series. Then integrate.

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2. What is the radius of convergence of this sequence?
Could we have predicted this radius of convergence from the beginning?
3. What is $G^{(137)}(0)$?