MAT1846 · TOPICS IN DYNAMICAL SYSTEMS
LENGTH AND LAPLACE RIGIDITY

Course information

Lectures	Mon 11:00 AM – 12:00 PM AP124
	Wed 11:00 AM – 1:00 PM AP124
Website	Quercus and Course home
Instructor	Jacopo De Simoi
Office	PG 201B
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Office Hours	Wed 1:00 PM – 2:00 PM

Online meetings

Due to the ongoing Omicron surge, a number of lectures will be delivered online. Please use the following zoom link to access the meetings for lectures and office hours.

University of Toronto recommended tech requirements for online learning can be found here. Lectures delivered online, including your participation, will be recorded on video and will be available to students in the course for viewing remotely and after each session. Course videos and materials belong to your instructor, the University, and/or other source depending on the specific facts of each situation, and are protected by copyright. In this course, you are permitted to download session videos and materials for your own academic use, but you should not copy, share, or use them for any other purpose without the explicit permission of the instructor. For questions about recording and use of videos in which you appear please contact your instructor.

Topics

The problem of spectral rigidity was popularized by M. Kac in 1976 as the question "Can one hear the shape of a drum?" and has been a very active research topic ever since. The problem, known as *Laplace rigidity* amounts to reconstruct a manifold, or a domain in $\mathbb{R}^n$, from the knowledge of the spectrum of its Laplace–Beltrami operator. The corresponding dynamical problem (known as *Length rigidity* or *Dynamical Rigidity*) consists in reconstructing a manifold, or a domain in $\mathbb{R}^n$, from the knowledge of the length of all periodic geodesics.

In this course we will introduce in some detail a selection of the most important known results from both the Laplace and the dynamical point of view. Depending on time and interest, we will review some of the following results:

- Wave Trace formula (Chazarain, Andersson–Melrose, Guillemin–Duistermaat 1976)
- Spectral rigidity of surfaces of negative curvature (Guillemin–Kazhdan 1978)
- Marked Length Spectrum Determination of surfaces of negative curvature (Otal 1990)
- Example by Vignerás, Sunada and Gordon–Webb–Wolpert (1980–1992)
- Laplace Determination of symmetric convex analytic domains (Zelditch 2008)
- Length Spectral Rigidity of convex domains (—, Kaloshin, Wei 2016)
- Local determination of smooth nonpositively curved Anosov Manifolds (Guillarmou–Lefeuvre 2018)

Prerequisites

The course will be accessible to graduate students with a basic knowledge of PDEs, differential geometry and real analysis. Prior knowledge in Dynamical Systems is optional.

Marking scheme

Students will be evaluated according to participation to lectures and a short presentation at the end of the term on topics related to the course.

Bibliography

Lecture notes will be provided for some topics, and a list of suggested readings will be kept up-to-date on the website; links to the relevant papers will also be posted on the website.